Asset Valuation

"*Price is what you pay. Value is what you get.*"

– WARREN BUFFETT.

1. INTRODUCTION

Understanding the causes of changes in the market prices of asset – financial and non-financial – is essential for corporate financial and operational decisions and household investments decision, as well as for interpreting financial markets and reading financial market signals as they apply to business activity, financial stability, and fluctuations in aggregate wealth and the wealth distribution. When we consider "asset" pricing we may have in mind equity prices. But, asset pricing in general applies to many classes of financial assets other than stocks – for instance, bonds and derivatives – and to non-financial assets such as gold and other precious metals, real estate, and raw materials, and even to collectibles like art, rare coins, antiques, etc. We will by default think of the assets as equity, since most of the pricing principles that apply to equity markets apply to other asset markets as well, but will highlight the instances in which pricing principles differ by asset class.

Generally, assets generate risky payoffs distributed over future time periods. The value of a particular asset can be viewed as the present value of the payoffs or cash flows, that is, the valuation of the cash flows *properly discounted for risk and time lags*. At a partial equilibrium level of analysis, both the risk-adjustment method and its quantitative value, as well as the time-lag adjustment method and value are taken as given. Often, however, we are also interested in general equilibrium models in which the proper risk and time adjustments are extracted endogenously.

Obtaining values for asset prices once payoffs are specified is just a small step from obtaining asset returns, and vice versa. For a given stream of direct payoffs and derived asset prices, the current one-period asset return is found as the sum of current price plus current payoff divided by previous-period price. Conversely, for given asset returns and given stream of direct payoffs, the asset prices can be backed out. In most instances it turns out to be more convenient to focus on expected and realized asset returns instead of asset prices. Expected returns provide valuable information, guiding capital budgeting decisions, portfolio choice, the evaluation of firm performance, and the rating of portfolio managers. Return realizations and expectations also provide insights in macro-economic business conditions, for instance because asset markets respond quicker than most other markets. By conventional use of terminology *valuation* concerns pricing, obtaining market price estimates of an asset taking the expected returns as given; whereas *asset pricing* somewhat obscurely does not pertain to pricing directly but involves determining expected returns.

There are various distinct methods for valuing an asset. A real asset can be valued based on: (1) the services the asset provides (the intrinsic value), (2) the price of like assets (the comparative or relative value), or (3) the cost of replacing the asset (the replacement value). An office building, for instance, may be valued by considering either the present value of the stream of actual or imputed rental income that it provides (in some cases, the scrap value or liquidation value of tearing the building down and selling the land and building materials); the price that a "similar" (in terms of location, design, state, and size) building was sold for in the recent past; or the cost of constructing the same building from scratch. The three evaluation approaches ideally would yield valuations that are close but this is not guaranteed. Financial assets, representing claims on real assets, can typically be valued along similar lines. For example, an initial public offering of equity would have as its intrinsic value the present value of its projected dividends; the comparative value would be the price of equity of a firm with similar risk and expected payoff characteristics; the replacement value approach involves a comparison to the cost of purchasing the equivalent of the issuing firm's tangible and intangible capital outright.

The price of a market-traded financial asset is directly available in which case valuation is obvious. However, the different valuation approaches provide useful inferences that differ by method. Contingent on the financial asset's actual market value, the intrinsic value method allows one to infer the return that the asset is expected to yield. The comparative value indicates if arbitrage profits or abnormal returns are (or, more commonly, were) available. The replacement value provides information on whether physical investment in a similar project is desirable.

In the current survey we examine several related accounting approaches for calculating the intrinsic value of financial assets. These approaches focus ostensibly on different "fundamentals" for obtaining intrinsic value but these fundamentals are related by accounting identities. For a given market price, the intrinsic value approach allows us to back out an expected return. We discuss more briefly at this point the comparative value approach, which provides exact values for derivative assets in absence of arbitrage opportunities or provides values for primary assets by comparing to expected returns of assets with comparable risk characteristics. For any financial asset, combining the intrinsic and comparative value approaches allows comparison of the expected return inferred from accounting information against the expected return based on the asset's risk, if a risk theory is specified. The replacement value approach can be related to Tobin's (1969) *q*-theory of investment which relates capital investment to the discrepancy between market valuation and book valuation of the investing firm.

In the remainder of the survey we discuss issues related to adjustment of cash flows that account for time preference in section 2, issues related to adjustment of cash flows that account for risk in section 3, and adjustment of cash flows to account for both in section 4. In section 4 we further discuss different valuation approaches and the distribution of value among the various stake holders in the asset. Section 5 examines the links of dividends and earnings to valuation. We introduce in section 6 the stochastic discount factor method of valuation. In section 7 we cover the relation between the various valuation approaches and expected returns, and section 8 provides applications and exercises.

2. ACCOUNTING FOR THE TIMING OF CASH FLOWS

Value must be adjusted for when cash flows are received. This time discounting topic is covered in finance texts under the heading of "the time value of money." The standard explanation for time discounting is that cash flows received earlier could be banked at a certain positive rate of interest and therefore are more valuable than cash flows received later. This is clearly the appropriate and simple basis for discounting later payments more. In the current section, however, we provide a more fundamental perspective that considers the economic forces establishing the appropriate time discount rate in equilibrium.

(a) The Subjective Rate of Time Preference

In a standard consumer choice model, we think of life-time utility U as determined by consumption levels c_i in the different T periods that the consumer has left to live $U = U(c_1, c_2, \ldots, c_T)$. Time preference (alternatively impatience, or discounting the future) is said to exist if future consumption, in a sense to be made precise in the following, is less valuable to the individual than is current consumption. Traditionally, explicit dynamic models have assumed time separable preferences, which imply a constant rate of time preference. However, other dynamic preference specifications, in which the rate of time preference changes over time are becoming increasingly popular. It is therefore important to consider a general definition for time preference that allows variation over time. Define the *discount rate* (or *rate of time preference* or *degree of impatience*) at time t as ρ_t. Then we can define the (one-period ahead) *discount factor* as $\beta_t = 1/(1+\rho_t)$. Thus, if the discount rate at time t is positive $\rho_t > 0$, then the discount factor is less than one, $\beta_t < 1$; if the discount rate increases (falls), the discount factor falls (increases).

The one-period discount factor at time t is defined exactly in a discrete-time formulation as:

$$(1) \qquad \beta_t(c_1, \ldots, c_{t-1}, c, c, c_{t+2}, \ldots, c_T) \equiv -\frac{dc_t}{dc_{t+1}}\Big|_{dU=0,\, c_t = c_{t+1} = c} = \frac{\partial U(c_1, \ldots, c_{t-1}, c, c, \ldots, c_T)/\partial c_{t+1}}{\partial U(c_1, \ldots, c_{t-1}, c, c, \ldots, c_T)/\partial c_t}.$$

The second equality follows since $dU = 0 = (\partial U/\partial c_t)\, dc_t + (\partial U/\partial c_{t+1})\, dc_{t+1}$. The discount factor, in words, is given as the *Marginal Rate of Intertemporal Substitution* between two points in time when consumption at the two points in time is equal. In intuitive terms it indicates, when initially $c_t = c_{t+1}$, how much of current consumption c_t the individual is willing to sacrifice for one extra unit of next-period consumption c_{t+1}.

For general preferences the discount factor may depend on consumption levels at all points in time.[1] For a *time-separable* intertemporal utility specification, however, utility can be written as:

$$(2) \qquad U(c_1, \ldots, c_T) = \sum_{t=1}^{T} \beta^{t-1} u(c_t).$$

The discount factor, then, can be obtained from the definition in equation (1) as:

$$(3) \qquad \beta_t(c_1, \ldots, c_{t-1}, c, c, c_{t+2}, \ldots, c_T) = \beta \equiv 1/(1+\rho).$$

Thus, the discount rate and factor are constant for the time-separable utility formulation.

Figure 1 shows that the discount factor can be obtained as the absolute value of the slope of the indifference curve along the 45° degree line emanating from the origin. The slope dc_t/dc_{t+1} should be smaller than one for positive time preference to exist. Time separability implies that the slope dc_t/dc_{t+1} does not depend on the level of consumption at times other than t or $t+1$. For the commonly used

[1] A typical misconception is that time variation in the discount rate implies time-inconsistent behavior. This is not the case as long as the form of the discount function in equation (1) does not change over time. That is, if the discount factor only depends on the stream of consumption and not explicitly on time [see Strotz (1956)]. For more, relatively non-technical, information about discounting see Price (1993).

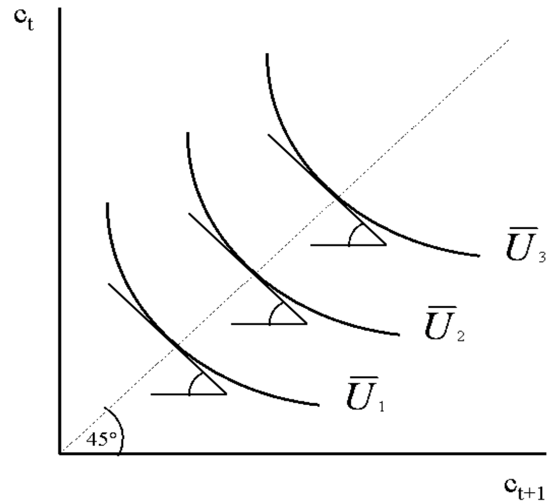

Figure 1
Time Preference
Utility is time-separable, as indicated by equal slopes along the 45° line.

time separable form in equation (2), the slope of dc_t/dc_{t+1} along the 45° degree line also does not depend on the level of $c_t = c_{t+1}$, which implies that the slope is the same for each indifference curve, as shown in Figure 1.

For positive time preference, it makes sense to discount future cash flows more than current cash flows. A natural discount factor for an individual valuing the payoffs of a particular asset would appear to be the subjective rate of time preference. However, we find in the following that, in a competitive market environment: (a) individual preferences should have no impact on the discount rate used in valuing an asset, and (b) even if preferences of all individuals are time-separable and identical, the proper discount rate in asset valuation is not generally equal to ρ.

(b) The Market Rate of Time Preference.

To examine time preference in a market context we take a look at the simple two-period world originally developed by the Classical economist Irving Fisher. Fisher (1930, Chapter 11) studied time preference by using the classical general equilibrium model but for a composite consumption good at two points in time rather than two different consumption goods at one point in time. The two-period context serves as a warm-up for more interesting infinite horizon dynamic models. As is standard in much of the traditional theory of finance we assume that markets are ***perfect***. This means (a) perfect competition and (b) no frictions. The latter assumes away

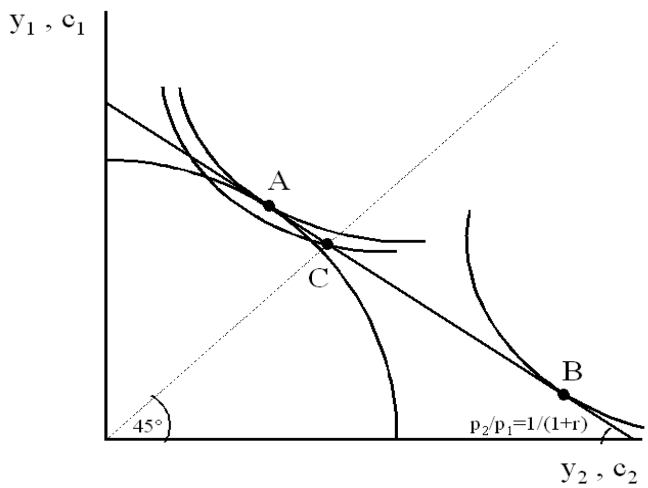

Figure 2
Objective Discount Factor and Fisher Separation
The objective rate of time preference will be the same for any two consumers. The objective and subjective rates of time preference will differ for the representative consumer.

transactions costs, borrowing constraints, short-selling constraints, taxes, etc. For now we also abstract from uncertainty. The equilibrium outcome of the model is displayed in Figure 2.

At point *A* we have the standard condition for a Pareto Optimal allocation sustained in competitive equilibrium by an equilibrium price ratio:

(4) $$MRIT(y_1, y_2) = p_2/p_1 = MRIS(c_1, c_2),$$

the Marginal Rate of Intertemporal Transformation is equal to the Marginal Rate of Intertemporal Substitution and this equality is supported in general equilibrium through the equilibrium price ratio p_2/p_1. This equilibrium price ratio represents the *market* (or objective) *discount factor,* generating the market rate of time preference. The graphical analysis here presumes that an aggregate production possibilities frontier and an aggregate preference ordering exist, or, similarly, that a representative firm and representative consumer exist. We can interpret

(5) $$p_2/p_1 = 1/(1+r),$$

where r is the real interest rate: saving one unit of consumption currently can buy you $1+r$ more units in the next period.

Valuation and time preference.

If we model the actions of a representative firm and consumer, then the two equalities in equation (4) can be derived from standard optimization problems. Assume that each household owns a firm that is endowed with a given initial endowment and requires no other inputs. The representative firm chooses output in each period to maximize the value of the firm V:

$$(6) \qquad V(y_0) = \underset{y_1, y_2, k}{Max} \; (p_1 y_1 + p_2 y_2) \; ,$$

$$\textit{Subject to}: \quad y_1 = y_0 - k, \qquad y_2 = f(k),$$

where p_1, p_2 are the consumption goods prices and y_1, y_2 the production quantities of the consumption goods in periods 1 and 2, respectively. Further, k represents capital (which equals investment here), y_0 the initial endowment, and $f(\cdot)$ is an increasing concave production function. The transformation curve in Figure 2 is the graph of the combined constraints from (6): $y_2 = f(y_0 - y_1)$. Substituting the constraints into the objective yields the first-order condition for optimal investment. Employing equation (5) then yields:

$$(7) \qquad f'(k) = p_1/p_2 = 1 + r,$$

marginal product of investment is set equal to the marginal cost of one unit plus interest sacrificed. The production levels y_1, y_2 can be determined from equation (7) and the constraints in (6).

To relate equation (7) to the $MRIT$, combine the constraints in equation (6) to eliminate x and totally differentiate to find that:

$$(8) \qquad MRIT(y_1, y_2) \equiv -\frac{dy_1}{dy_2} = \frac{1}{f'(k)} = p_2/p_1 \; ,$$

thus showing that equation (7) produces the first equality in equation (4).

The representative household, with its only source of income the profit of the representative firm, maximizes:

$$(9) \qquad \underset{c_1, c_2}{Max} \; U(c_1, c_2) \; ,$$

$$\textit{Subject to} \;\; p_1 c_1 + p_2 c_2 = V .$$

The first-order condition for the household becomes:

$$(10) \qquad \frac{U_1(c_1, c_2)}{U_2(c_1, c_2)} = p_1/p_2 = 1 + r,$$

where the subscripts on the utility function indicate partial derivatives. In the case of time-separable utility described in equation (2) we obtain an equation that shows up frequently in the literature:

$$(10') \qquad \frac{u'(c_1)}{\beta u'(c_2)} = 1 + r.$$

Whereas here the subjective gross discount rate $1 + \rho \equiv 1/\beta$ is equal to the market discount rate, whenever $c_1 = c_2$, this is not generally the case.

Totally differentiating an indifference curve gives the *MRIS* :

$$(11) \qquad MRIS(c_1, c_2) \equiv -\frac{dc_1}{dc_2} = \frac{U_2(c_1, c_2)}{U_1(c_1, c_2)} = p_2/p_1,$$

yielding the second part of equation (4).

Implications

Consider the preferences of an individual consumer, taking market prices as given. In Figure 2, view the allocation *A* as that for a representative consumer *a*. For a consumer *b* that differs from the representative consumer *a* but has an identical endowment (they own identical firms), the optimal allocation may be found at point *B*. Note that $MRIS_A = MRIS_B$; this fact illustrates our first result of interest, namely:

> **RESULT 1 (FISHER SEPARATION).** *In a perfect market without uncertainty individual preferences are irrelevant for the discount factor guiding individual decisions, including the decision how to value a particular asset.*

The Fisher Separation result, based on Fisher (1930, Chapters 6-8), also applies in a corporate finance situation where the way to maximize the value of the firm to shareholders should be independent of individual preferences but instead should be based on market prices, such as the market discount rate. The argument is that any feasible trade for the individual must lie along the intertemporal budget line. Any allocation along the budget line that differs from point B of course will change the realized marginal rate of intertemporal substitution but will also move the individual to a lower indifference curve. Both agents a and b, holding stock in similar firms, are better off if the firms ignore their time preferences!

A second result is apparent by considering point C in Figure 2 which is the intersection between the market discount rate line (also the budget line for the representative consumer) and the 45° line. It indicates also the subjective discount factor of the representative consumer, given as the slope of the indifference curve at point C. If the preferences are intertemporally separable then this slope is equal to the constant β and is clearly different from the slope of the budget line. Thus:

RESULT 2 (MARKET AND SUBJECTIVE DISCOUNT RATES). *In a perfect market without uncertainty, market and subjective rates of time preference generally differ, even for the representative consumer.*

When preferences are intertemporally separable as in equation (10′) the issue is clearest. The market discount factor is given by $MRIS(c_1, c_2) = \beta u'(c_2)/u'(c_1)$ which equals $1/(1+r)$ and differs generally from β which equals by definition $1/(1+\rho)$, since the intertemporal production tradeoffs implied by the *MRIT* may easily determine an allocation where c_1 and c_2 are not equal. Hence, the representative consumer discounts cash flows differently (namely by the constant market discount factor) from the way she discounts utility, which for intertemporally separable preferences is discounted by the constant subjective discount factor. In simple – albeit somewhat imprecise – terms one may say that *wealth* is discounted differently from the way *utility* is discounted.

An alternative way of seeing how the real interest rate is determined in the Fisher model is to equate savings and investment. Equation (7) and the concavity of the production function show that investment is a negative function of the real interest rate. Set the numeraire $p_1 = 1$ and define savings s as total income left for future spending: $s = p_2 c_2 = c_2/(1+r)$. Equation (10) and the budget constraint in (9) then imply a savings schedule for given income V. The slope of the schedule is difficult to determine because the real interest rate has both a substitution effect and an income effect on saving: a higher interest rate implies a higher direct return to saving, leading to more future consumption, but the lower effective cost of future consumption also means that less savings are needed to finance a given level of consumption. The net effect is not clear. If we assume the income effect is less important than the substitution effect then the savings schedule has a positive slope and the real interest rate is found at the point where savings and investment are equal.

(c) Present Value and Value Additivity

We have seen that in the certainty context of the Fisher model we can use the real interest rate to discount future consumption streams. Take now a real cash flow to be received (with certainty) at a time s periods in the future. What is this future receipt worth now? Or, in other words, what is its present value? We obtain the correct answer in two different ways. First, using the ideas of opportunity cost and arbitrage. By postponing the cash flow by s periods you forego interest. The Future Value at time s of an initial investment I_0 is given as $FV_s = (1+r)^s I_0$, where we account for periodic compounding and we assume a constant real interest rate. To find the current value of a cash flow CF_s to be received at time s and of equal value to FV_s, from arbitrage, the Present Value at time 0, V_0, must be equal to I_0 , so that we can invert the $FV_s = (1+r)^s I_0$ equation to obtain

$$(12) \qquad V_0 = \frac{CF_s}{(1+r)^s} .$$

In a perfect market environment, the opportunity you sacrifice – your next best alternative used for calculating the opportunity cost – must be economically identical to the project you evaluate. Given that the return on this best alternative is equal to r , we can similarly interpret the present value calculation as accounting for the appropriate opportunity cost: the cash flows of a project should be valued such that both this project and the alternative generate the same return.

As a second, related, approach to calculating present value, one could extend the logic of the Fisher analysis to a multi-period context. Assuming a constant objective discount factor we then from equation (5) have the discount factor as $p_s/p_0 = (p_s/p_{s-1})(p_{s-1}/p_{s-2})...(p_1/p_0) = [1/(1+r)]^s$. This is the appropriate factor to discount cash flows, yielding again equation (12).

In the perfectly competitive Fisher world it follows that each component of a stream of cash flows could be bought or sold separately. Thus, the present value of a stream of cash flows is just the sum of the present values of the separate cash flows. This property is called *Value Additivity:*

> **RESULT 3 (VALUE ADDITIVITY).** *In perfect capital markets, the value of an asset representing a series of cash flows is equal to the sum of the values of these cash flows valued separately.*

Accordingly, we can write the present value of a stream of future payments as:

$$(13) \qquad V_0 = \sum_{s=1}^{\infty} \frac{CF_s}{(1+r)^s} .$$

In practice, Treasury Bonds are broken up into their separate components: coupon payments and the zero-coupon remainder (called STRIPS, an acronym for Separate Trading of Registered Interest and Principal of Securities). As dealers find it worth their while to incur the transaction costs to do this, it appears that Value Additivity holds only approximately in actuality. Value additivity implies also that mergers should yield no financial synergy. The basic reason is that, under the perfect market assumption, any security holder is already able, on his own account, to buy the securities of the merging firms.

(d) The Gordon Growth Model

The cash flows in equation (13) could follow any pattern. In many instances they will be zero beyond some point *T*. In many cases cash flows can only be approximated and one has but vague ideas of its pattern over time. These vague ideas are often best summarized by an initial value, that is quite precisely known, and an average growth rate, maybe less precisely known.

The *Gordon Growth Model* of Gordon and Shapiro (1956) and Gordon (1959, 1962) makes the practical assumptions of a known initial cash flow CF_1 and a known constant growth rate g. It is a useful model not only because it provides a reasonable approximation to the entire future of cash flows, it also generates a simple expression for the Present Value. With these assumptions, equation (13) becomes:

$$(14) \qquad V_0 = \sum_{s=1}^{\infty} \frac{(1+g)^{s-1} CF_1}{(1+r)^s} = \frac{CF_1}{r-g} .$$

That the second equality holds is easy to see by realizing that we have an infinite geometric series with a constant geometric factor $(1+g)/(1+r)$ that declines under the reasonable assumption that $g < r$. Clearly, if cash flows grow at a higher or equal rate compared to the real rate of interest, $g \geq r$, the present value is infinite as the present value components do not decline over time.

Equation (14) can be quite useful in a variety of cases. One application is obtained if we apply the Gordon Growth Model to the overall stock market. Strictly taken the model only applies for a fairly certain growth rate. However, as we find later in this survey, one may just discount expected cash flows for risk by discounting by the appropriate expected real stock return (the proper opportunity cost for the riskiness of this particular project) rather than the real interest rate.

The only cash flows generated by stocks in the aggregate are the dividend payments since we can ignore the pure distribution effects rendered by capital gains or losses from stock trading. Thus, the value of a stock should be the present value of the stream of dividends generated. In equation (14) we may thus interpret CF_1 as a typical annual aggregate dividend payment and V_0 as the current price of a stock index. The *price-dividend ratio* is then equal to $V_0/CF_0 \approx V_0/CF_1$. If average price-dividend ratios for S&P 500 firms equal around 33 then we can infer from equation (14) that $r - g = 0.03$. Historically, real stock returns have been around 0.08 (8%). Hence, stock valuations imply that $g = 0.05$ – that dividends growth is expected on a permanent basis to equal 5%. This number substantially exceeds the around 2% historical growth rate of real per capita GDP and the similar projections for its growth. Explanations may be that the growth of profitability on a permanent basis is expected to exceed real growth of GDP; that the risk premium inherent in stock returns has fallen permanently; that we have a permanently altered economy; or that stocks are simply overvalued.

Comparing the impact of changes in cash flow and the discount rate

If fluctuations in fundamentals are the main determinants of changes in market prices then we can ask whether fluctuations are more likely due to cash flow news in the numerator of equation (14) or to discount rate news in the denominator of equation (14). Empirically, discount rate fluctuations must be an important source of price variability to explain observed variability as stemming from changes in fundamentals rather than mispricing. Taking derivatives in equation (14) allows a useful perspective on this issue. If market price equals intrinsic value, $P_0 = V_0$, then from equation (14) the immediate percent return from a permanent one percent increase in cash flows is

$$(15) \qquad d\ln P_0 / d\ln CF_1 = 1.$$

So a permanent one percent increase in the *level* of dividends raises market price by one percent.

On the other hand, a discount rate permanently higher by one percentage point, $d(1+r) \approx dr = 1$, implies from equation (14) that

$$(16) \qquad d\ln P_0 / dr = \frac{-1}{r-g} = -P_0 / CF_1.$$

Hence, for equity a one percent higher discount rate implies a capital loss approximately equal the the price-dividend ratio. Given the numbers discussed above this corresponds to a 33% loss. It follows that even minor fluctuations in discount rates may be important determinants of equity price fluctuations. Note that the effect of a permanent increase in the *growth rate* of dividends g by one percent is of equal but opposite magnitude $d\ln P_0/dg = P_0/CF_1$, translating to a 33% capital gain.

(e) Inflation and Valuation

A long-standing empirical observation is that stocks are not a perfect hedge for anticipated inflation. See, for instance, Jaffee and Mandelker (1976) and Fama and Schwert (1977). Inflation, anticipated as well as unanticipated, appears to have a clear negative impact on real stock returns. Here we see however that theoretically, as a first approximation, inflation should have no impact at all on stock prices and returns.

Assume that future dividends in real terms are unaffected by inflation π. This is a reasonable assumption since firm profits in real terms should not change–both costs and revenues ought to change proportionately with inflation, causing nominal profits to rise at the rate of inflation and real profits to be unchanged. Continuing with the information of the present value calculation in equation (14) for simplicity, it is assumed that real cash flow X_l is not affected by inflation and thus equals $(1+\pi)\,CF_l$ in nominal terms. Due to inflation, nominal dividends will be growing at a higher rate g_n and we now discount the nominal cash flows by the nominal return i. Equation (17) then gives the valuation for the Gordon Growth Model when inflation is introduced:

$$(17) \qquad V_0 = \sum_{s=1}^{\infty} \frac{(1+g_n)^{s-1}(1+\pi)\,CF_1}{(1+i)^s} = \frac{(1+\pi)\,CF_1}{(1+i)-(1+g_n)} = \frac{CF_1}{r-g}\,.$$

The second equality follows by the definitions of real interest rate r: $(1+i)/(1+\pi) = 1+r$ and real growth rate g: $(1+g_n)/(1+\pi) = 1+g$. It follows in the context of our assumptions that: (a) current stock prices are in theory not affected by anticipated inflation, and that (b) one may alternatively discount nominal payoffs with the nominal interest rate or discount real payoffs with the real interest rate.

Empirically, it appears that both anticipated and unanticipated increases in inflation lower stock prices, so that stocks appear to be poor inflation hedges. This is clearly counter to our discussion which implies that anticipated inflation's effect on stock prices should be neutral. The most prominent rational explanation for this apparent non-neutrality is by Fama (1981): since stocks react quickly to new information, they decrease on news of a business cycle downturn, before this downturn happens. But a downturn in activity, given constant money growth, implies higher inflation (in a classical macro model). Thus, higher inflation is associated with (but does not cause!) lower stock prices; which is what we find empirically.

Modigliani and Cohn (1979) argue that in practice agents are often not clear about discounting nominal/real payoffs with nominal/real interest rates. A common mistake is to use actual (i.e., nominal) interest rates to discount payoffs but to consider an inflation-adjusted (i.e., real) growth rate for payoffs. This type of money illusion implies directly that anticipated inflation negatively affects stock prices. In a subsequent section we discuss the implications of the Modigliani-Cohn hypothesis for asset returns.

(f) Remaining Issues in Time Discounting

Even for deterministic cash flows, the previous discussion has not exhausted the issues that may arise in valuing payments that are delayed. One issue is liquidity. There should be a substantial difference in valuing a future cash flow of an asset that is highly marketable as compared to an asset that could not be sold before the payment is due. For the latter type of asset (such as your own human capital, or software in development), the liquidation value is important. In case of a personal emergency, when the project has to be abandoned for whatever reason, its liquidation value matters. In a perfect market, of course, *individual* liquidity risk is not an issue.

Market liquidity risk matters, however, even in a perfect market. If the asset is riskless and could be sold for true value on short notice, the proper opportunity cost is the rate of interest of a short-term riskless bond, such as a T-Bill. On the other hand, the opportunity cost of holding an asset whose intermediate value may vary is more like the rate of return on a long-term riskless bond, such as a Government Bond. Based on the term structure of interest rates literature, the long-term bond rate usually exceeds the short-term bond rate, so that the less liquid asset will be worth less.

A further complication for time discounting is related to variability of the opportunity cost, the discount rate. If the varying levels of the discount rate are known or predictable, this is not a big issue, the discount factor will just differ from period to period. An uncertain opportunity cost, however, is a different matter, especially if the opportunity cost is correlated with the cash flow. It seems then that, our simple time-discounting method requires not only known cash flows but also a known pattern of the opportunity cost in order to be strictly valid.

3. ACCOUNTING FOR RISK

Uncertainty that we can somehow quantify is referred to as risk. Most individuals dislike risk, at least when it is quantitatively large enough to potentially put a serious dent in one's wealth and limit consumption choices. In this section we discuss the concept of risk and how it is applied to adjust future risky cash flows for the undesirability of fluctuations in their realization.

(a) One-Sided and Two-Sided Risk

The standard use of the concept of risk by finance academicians relates to a *two-sided* risk: it considers the impact of losses as well as gains, relative to expectation. This concept of risk was formalized precisely by Rothschild and Stiglitz (1970). For a risky cash flow, they define an *increase in risk* as adding independent noise to the cash flow for a *given expected value* of the cash flow, which, they show, is equivalent to putting more probability mass in the tails of the cash flow distribution. They prove that any risk averse investor (that is, one who has concave utility over consumption) will dislike such an increase in risk. (In fact they show that, *only* for such a definition of risk, any risk averse investor dislikes more risk)[2].

The Rothschild-Stiglitz definition of risk is an intellectually satisfying one. However, it is of little practical use in pricing assets for the following reasons. First, it provides only a partial ordering of risky prospects. That is, many risks cannot be compared in this way since one can not separate out an independent noise by which these risks differ. Thus, some investors would prefer the one risk, some would prefer the other risk. Second, even if two risks can be ranked in the Rothschild-Stiglitz way, this is only an ordinal ranking and it is difficult to put a cardinal number on the difference for pricing purposes. Third, this general definition of risk is individual-based and ignores the Fisher Separation result that applies for competitive markets. Thus, in the following we resort to more operational definitions of risk. Conceptually, however, we continue to think of risk in these general terms. The different aversions by individuals to a particular asset's risk, in the aggregate implies a discount to the asset price and a premium to the asset's expected return which is market determined.

In the business community, the implicit concept of risk is often a *one-sided* one. "Putting your money at risk" means that you may lose it. It doesn't really speak to what happens when you gain. This concept of risk is associated with *default risk* (as opposed to the two-sided *market risk*)

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[2] This concept of risk considers an increase in payoff variability for a given mean payoff and is often referred to as a *mean preserving spread.* A slight generalization is the concept of *second-order stochastic dominance* developed independently by Hadar and Russell (1969) and Hanoch and Levy (1969). A "less risky" payoff distribution second-order stochastically dominates a "more risky" payoff distribution if all risk averse individuals prefer it. This generalizes the Rothschild-Stiglitz concept of risk since it may occur not only if the second payoff distribution is a mean preserving spread but also if the mean of the second payoff is not preserved and is lower.

and occurs in its purest form in a risky bond or junk bond. It differs from the academic definition of risk in that it matters even for a risk-neutral individual. It further is difficult to work with since introducing one-sided risk implies a change in the mean of the payoff as well as in the variance of the payoff. In the following we always employ the two-sided concept of risk. We show next how to take one-sided "risk" into account.

Assume that the market is risk neutral so that two-sided risk is irrelevant (in exercise 4 of section 8 we drop the assumption of a risk neutral market). Imagine a firm issuing a perpetual bond with constant annual coupon payment X and a probability of bankruptcy of F. When the firm goes bankrupt no further coupon payments will be paid. What is the value of this perpetual bond? Given a risk-neutral market, all we need to do is to calculate the expected cash-flow for each period and discount it by the risk free interest rate. The expected cash flow for, say, period n is easily calculated as $E(CF_n) = (1-F)^{n-1}X$. Thus, the (present) value V of the perpetual bond equals:

$$(1) \qquad V = \frac{X}{1+r} + \frac{(1-F)X}{(1+r)^2} + \frac{(1-F)^2 X}{(1+r)^3} .... = \frac{X}{r+F} .$$

The second equality holds along the lines of the Gordon Growth Model derivation but with F replacing $-g$. Equation (1) implies that the probability of default is added to the discount rate: you discount payments further in the future, both because of impatience and because you are less likely to receive them.

Although seemingly obvious, the return implications of equation (1) are sometimes poorly understood. The *unconditional* expected return from owning the perpetual bond under the assumption that its market price is the present value is given as

$$(2) \qquad E(r) = \frac{X - FV}{V} = r .$$

The first equality follows from the fact that the payment is X and that there is no capital gain unless the firm goes bankrupt, with probability F, at which point the gain is $-V$. The second equality is a restatement of equation (1). Equation (2) simply reflects the fact that there is no risk premium for this perpetual bond. Although there is uncertainty about the payoff the risk carries no premium in the market because it can, supposedly, be diversified by holding a portfolio of bonds with unrelated bankruptcy probabilities. In a fully diversified bond portfolio the fraction of bonds lost due to bankruptcy will equal the (average) bankruptcy probability F, and the return equals exactly r, the risk free rate.

Conditional expectations and survivorship bias

More interestingly, consider the *conditional* expected return, conditioned on the event that the firm does not go bankrupt during the period. Then

(3) $$E(r\,|\,survival) = X/V = r + F\ ,$$

as follows directly from equation (1). The possibility of default lowers the market price of the bond, If default does not occur so that the full coupon payment is received, the payoff is relatively large as a fraction of the purchase price and the return exceeds the risk free rate. The extra return of F looks like a risk premium but it is not; it arises only because we do not calculate the appropriate expected value given in equation (2).

Equation (3) illustrates the concept of *survivorship bias*. In many instances asset return data collected, exclusively involve assets that have survived over the years.[3] With the hindsight of knowing that the asset has survived, its average return provides a biased estimate of the true expected return. For the perpetual bond the average returns becomes $r + F$ suggesting a risk premium above the risk free rate. However, the high average return arises merely because in the average we ignore the low returns of those bonds that defaulted. The bonds that did not survive to pay coupon payments also did not survive to make it into the data set.

In summary, one-sided "risk" may be taken into account by simply adjusting the *expected value* of the cash flow. Clearly, a higher probability of default lowers expected cash flows by more the further in the future they are expected to be received. This affects therefore investors even in a risk neutral market. In a realistic risk averse market situation, one-sided "risk" may still be riskless according to certain asset pricing models if it is uncorrelated with market returns and therefore disappears in a well-diversified portfolio. Alternatively, if we do not want to adjust the expected value of the cash flow to account for default risk, equation (1) shows that we may be able to augment the risk free discount rate with the default probability to account for one-sided risk. Here the cash flow naturally should not be interpreted as the expected cash flow but as the promised cash flow.

(b) Risk Adjustment in Discounting

Suppose that we want to value a risky payment to be received one period from now. First

[3] One standard example of how a survivorship bias arises is when data collection starts with a full set of firms at a particular point in time and then works backwards, "backfilling" the data, to obtain the historical data for this set of firms.

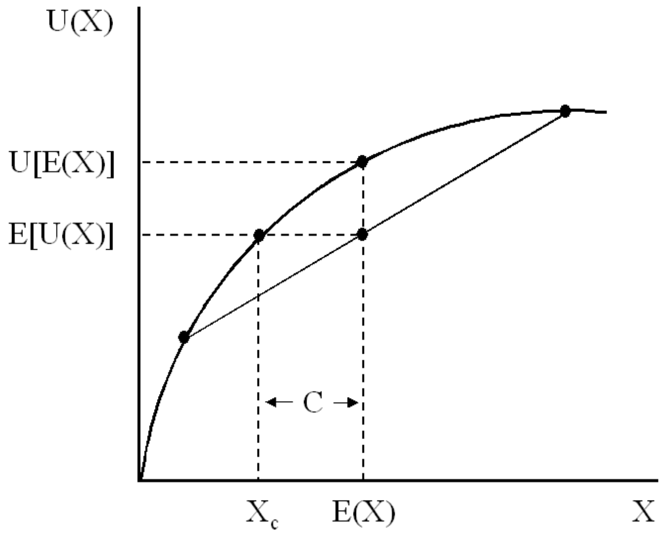

Figure 3
Risk Adjustment
For a two-point distribution, the amount, X_c, is the certainty equivalent amount. The risk adjustment or insurance premium needed to obtain X_c is given by the interval C.

we address the issue from a purely theoretical perspective: What compensation C would an individual require to be indifferent between the risky cash flow X and a certain cash flow with the same expected value $E(X)$? Using the standard expected utility criterion we find C from:

$$(4) \qquad E[u(X)] = u[E(X) - C].$$

Figure 3 shows graphically how to obtain C given a two-point distribution for X. Note that the risk introduced here is a specific example of a Rothschild-Stiglitz increase in risk. Once we have the value for C we could obtain the appropriate discount rate μ_X for X from the following:

$$(5) \qquad \frac{E(X)}{1 + \mu_X} = \frac{E(X) - C}{1 + r} .$$

We then find μ_X directly from equation (5) as:

$$(6) \qquad \mu_X = r + \frac{C(1 + r)}{E(X) - C} ,$$

where the term after the "+" sign is the risk premium in the "required return" for X.

Adjusting cash flows for risk in this manner is subject to similar problems as the use of the Rothschild-Stiglitz definition of risk for this purpose: It is difficult to apply, and it ignores the Fisher Separation result. Notice for instance that C differs by individual and depends generally on initial wealth, even for otherwise identical individuals. Next we consider the alternative, the standard market-based approach for discounting risky cash flows.

A market-based approach is based on the celebrated CAPM (Capital Asset Pricing Model). According to the CAPM the expected return of any asset x is determined according to

$$(7) \qquad \mu_x = r + \lambda \, \mathrm{Cov}(r_x, r_m) \; ,$$

in which the covariance term indicates the covariance between the return on asset x and the return on the overall market, asset m. It reflects the fact that only "systematic" risk that is correlated with the market matters since other "non-systematic" risk may be diversified away. Further, λ indicates the "price of covariance risk" which is determined at the market level.[4]

One may now use the "arbitrage" principle together with the opportunity cost idea to price the cash flow X: if one were to invest amount I in asset X', with known expected return μ_X, the expected forward value would be: $E(FV^{X'}) = I\,(1 + \mu_X)$. Assuming that X' has the same risk characteristics as asset X we can then set the required return for X equal to the expected return for X': $\mu_{X'} = \mu_X$. Then equating the expected forward value of X' to the expected cash flow of X we obtain the (present) value of asset X as:

$$(8) \qquad V^X = \frac{E(X)}{1 + \mu_X} \, .$$

Thus, discounting should be accomplished by using the expected return on a "similar" asset as the discount rate. Of course, what is similar depends on the particular asset pricing model. For the CAPM, "similar" means for the returns of both assets to have the same covariance with the market. If we apply the CAPM equation in (7) to equation (8) we can obtain the ***certainty equivalence*** expression for the value of asset X, as follows.

Bear in mind that the return on asset X is defined as $r_X = X/V^X$ given that the asset is priced

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[4] The equivalent, better known expression for mean return in the CAPM is $\mu_x = r + \beta_x(\mu_m - r)$ in which the "beta" is $\beta_x = \mathrm{Cov}(r_x, r_m)/\mathrm{Var}(r_m)$ and the "market risk premium" is $\mu_m - r = \lambda\,\mathrm{Var}(r_m)$

correctly. Then we can write $\mathrm{Cov}(r_X, r_m) = \mathrm{Cov}(X, r_m)/V^X$ (see Appendix C for the linearity property of the covariance operator). Applying equation (7) to equation (8) and solving for V^X based on this covariance expression we obtain:

$$(9) \qquad V^X = \frac{E(X) - \lambda\,\mathrm{Cov}(X, r_m)}{1 + r} \, .$$

Thus, one may account for risk by using the appropriate "opportunity cost" discount rate. Or, under the assumptions of the CAPM, one may adjust the expected cash flow for the systematic risk inherent in the cash flow to yield the certainty equivalent value of the expected cash flow and then discount using the risk free rate.

There are aspects of accounting for risk in multiple-period present value calculations that we have not yet addressed and which are quite complicated, even in the context of the CAPM. These aspects are related to cross-period correlations in cash flows and to uncertainty about discount rates.

4. INTRINSIC AND ALTERNATIVE VALUATION METHODS

The present value discussed in the previous will be called either "intrinsic" value or "fundamental" value in the following (and in the literature is sometimes also referred to as "substantial" value) when we are dealing with an asset or group of assets rather than a single project. We start by establishing some basic results that apply to intrinsic valuation and its relation to market prices.

(a) Present Value, Intrinsic Value, and Market Efficiency

We can write the intrinsic value of a company or any asset as follows:

$$(1) \qquad V_t = \frac{E_t(CF_{t+1} + V_{t+1})}{1 + \kappa_t} \, .$$

The subscript t on the expectations operator $E_t(\cdot)$ represents expectations conditional on all available information at time t; CF_t represents cashflows distributed to the stakeholders at time t; κ_t indicates the ***required return*** at time t that prevails for this asset over the upcoming period $t + 1$. The required return is the average return that investors require, in the current market environment, to compensate for the return risk (and time preference) of investing in the asset. Thus, the present value of the firm can be viewed as the properly discounted value of the expected "payoff" received in the upcoming period: the cashflow generated plus the remaining value of the firm at that time.

The intrinsic value V_t need not equal the market value P_t. Paraphrasing the statement of value-investment icon Warren Buffett (opening quotation): you do not always get what you pay for. However, a common way (for instance see Summers, 1986). of operationalizing the idea of ***market efficiency*** is to define it as follows: a market is efficient when the market price at all times equals the intrinsic value of the asset. Note that the intrinsic value expression in equation (1) assumes that *all available information* is used in forming expectations about future payoffs. When the market price is equal to this value, and so the market is by definition efficient – no easy profit opportunities are available from asset trade. The issue of market efficiency becomes a bit more complicated after introducing imperfect information issues, but here we take an efficient market to be one where, at every market date, (market) price = (intrinsic) value.

How does the average return from investing in an asset relate to the required asset return? The answer depends on whether the equity market is efficient. Suppose for instance that $P_t = \eta_t V_t$ in each period with $E_t \eta_{t+1} = 1$. We obtain the ***expected return*** as $\mu_t \equiv E_t r_{t+1} \equiv E_t [CF_{t+1} + (P_{t+1} - P_t)]/P_t$. It consists of the current yield plus the capital gain. From equation (1) then $1 + \kappa_t = \eta_t (1 + \mu_t)$. In contrast, if $P_t = V_t$ in each period it must be that $\kappa_t = \mu_t$. To summarize this (tautological) fact:

> **RESULT 4 (EXPECTED RETURN IN AN EFFICIENT MARKET).** *If the market is* efficient *in the sense that market price equals intrinsic value*, $P_t = V_t$, *then the expected return equals the required return*, $\mu_t = \kappa_t$.

Another way of stating the same result is to say that the ***abnormal return***, $r_{t+1} - \kappa_t$ (the realized return $r_t \equiv [CF_{t+1} + (P_{t+1} - P_t)]/P_t$ net of its opportunity cost κ_t), is zero on average.

In general, researchers have two choices for modeling asset markets. First, the equilibrium or *efficient markets* approach. With $P_t = V_t$ by construction, price fluctuations not explained by changes in expected dividends are automatically attributed to changes in the required return. When changes in market conditions are anticipated, required returns typically change and because $\kappa_t = \mu_t$ expected returns are predictable even though markets are efficient by definition. Fama and French (2002) provide an example of the use of intrinsic value in an efficient markets context by engaging the Gordon Growth Model to identify expected market returns.

The second modeling choice is a disequilibrium or *inefficient markets* approach in which

$P_t = \eta_t V_t$. See Lee, Myers, and Swaminathan (1999) for a nice example of this approach. Return predictability here is not (necessarily) due to changes in required returns but is likely the result of assets being "overpriced" or "underpriced". The difference between the approaches is not one of semantics. If an expected excess return is high because the required excess return is high then the asset is not necessarily more desirable: the higher expected return reflects an increase in the risk premium. If an expected return is high because the asset is underpriced then the asset is more desirable since is provides a higher expected return even though the required return may be unchanged.

Because the equilibrium approach is prevalent in the academic literature, the terms "required return" and "expected return" are often used interchangeably. In the recent literature the term "required return" seems to be utilized less frequently than in the past. However, to deal explicitly with the possibility of mispricing it is convenient to continue using it. We will understand by *required return* the average return that is necessary in market equilibrium to compensate investors for the risk inherent in the asset. The required premium depends naturally on the maintained risk-premium theory.

The discounted cash flow formulation

Solving equation (1) forward produces the standard discounted cash flow representation:

$$(2) \qquad V_t = \frac{E_t CF_{t+1}}{(1+\kappa_t)} + \frac{E_t CF_{t+2}}{(1+\kappa_t)(1+\kappa_{t+1})} + \frac{E_t CF_{t+3}}{(1+\kappa_t)(1+\kappa_{t+1})(1+\kappa_{t+2})} + \dots ,$$

given that a transversality condition $\lim_{s\to\infty} [\, E_t CF_{s+1} / \prod_{i=t}^{s} (1+\kappa_i)] = 0$ holds, and with the assumption that the required returns are deterministic. Equation (2) calculates value as the present value of all expected future cashflows. Equations (1) and (2) are conceptually similar (identical if the transversality condition holds and required returns are deterministic) ways of specifying intrinsic value.

The certainty-equivalent discounted cash flow formulation

Alternatively we can obtain the discounted dividend representation in certainty-equivalent form, in which we can first adjust the expected cash flows for risk and then discount by the risk free rate. The certainty-equivalent discounted dividend representation is

$$(3) \qquad V_t = \frac{E_t CF_{t+1} - (\kappa_t - r_t^f) V_t}{(1+r_t^f)} + \frac{E_t CF_{t+2} - (\kappa_{t+1} - r_{t+1}^f) E_t V_{t+1}}{(1+r_t^f)(1+r_{t+1}^f)} + \dots ,$$

where r_t^f is the risk free (discount) rate for time period t which is assumed known but may be time varying. It is easy to check that equations (2) and (3) are interchangeable: in equation (3) just multiply V_t on the left-hand side by $1 + r_t^f$ and bring the V_t term in the first payoff on the right to the left; then repeat by multiplying V_t by $1 + r_{t+1}^f$ and bring the V_t term in the second payoff on the right to the left, etc. to obtain equation 2). So the risk adjustment amount applied to adjust expected cash flows consists of the appropriate risk premium $\kappa_t - r_t^f$ applied to the value of the capital tied up, V_t. Of course, it is also possible to apply the certainty-equivalence adjustment to equation (1) and solve forward to generate equation (3).

The Gordon Growth Model with uncertainty

The Gordon Growth Model under uncertainty is a special case of the general formulation in equation (2). If we assume a constant required return, $\kappa_t = \kappa$ for all t, and a constant growth rate of expected cashflows, $CF_{t+1} = (1+g)\,CF_t$ for all t, then:

$$(4) \qquad V_t = \frac{E_t CF_{t+1}}{\kappa - g} .$$

We compare the intrinsic value to other valuation concepts. Apart from intrinsic value (equivalently, fundamental value, substantial value, or present value) common alternative value indicators are: *book value*, *market value*, *value of capital*, and *liquidation value*. We further examine to what extent these measures reflect *growth options* embedded in the asset and how well value is identified by financial ratios such as the price-dividend ratio and the earnings yield. In addition to valuation of the firm as a whole we examine the basis for distribution of value across the financial claimants of the firm – equity and debt holders in particular. To these ends we start by presenting a simple accounting system that fixes ideas and facilitates understanding of the relations among the valuation concepts.

(b) A Financial Accounting System

Consider a highly stylized accounting system that we may apply to any firm. The firm has a capital stock financed by debt and equity; the firm's income after wage payments in each period depends on the capital stock; the firm finances investment to maintain and expand the capital stock, and the remainder of available income is distributed to debt and equity holders. We maintain the following simplifying assumptions:

1. Capital (K_t) consists of three types: working capital (cash and other financial assets),

tangible capital (buildings, equipment, inventories), and intangible capital (patents and other output from research and development (R&D), goodwill and other output from marketing). Thus, gross investment (I_t) includes increases in cash holdings, R&D, as well as more traditional components such as the cost of replacing machinery and expanding production capacity. Each type of capital is assumed to depreciate at the same rate (δ), be it through inflation, depreciation, or amortization.

2. Debt is assumed to be risk free. One may think, for instance, of tangible capital as serving as collateral for the debt. It follows that: (a) the book value of debt (B_t^D) equals the intrinsic value of debt (V_t^D), and (b) the required return on debt (κ_t^D) equals the risk free rate (r^f) which is further assumed to be constant over time.

3. All changes in the value of the assets are reported through the income statement. This leads to a system of *clean surplus accounting* as discussed by Ohlson (1995) and Feltham and Ohlson (1995), according to which the retained earnings that contribute to the book value of equity (B_t^E) are obtained as net income (NI_t) from the income statement minus dividends.

4. Taxes payments and tax shields are ignored. See Fernandez (2004, 2009) or Oded and Michel (2007) for a discussion of accounting for taxes in the context of different valuation methods.

Under these assumptions we present the income statement, the cash flow statement, and the balance sheet (the balance sheet in accounting terms as well as marked to market) in Table 1. First define some terms. The cash flow related to equity finance equals the dividends paid to current stockholders (D_t) minus revenue from new issues of stock net of the value of stock repurchases (NB_t^E). The net cash flows to debt holders equal the interest payments on the debt (RP_t) minus the new debt issued during the period (NB_t^D). The operating cash flow, the net cash flow from operations, $Y_t - WP_t$, is also known as $EBITDA$ (Earnings Before Interest, Taxes, Depreciation, and Amortization); and operating profit, the net cash flows from operations after writeoffs (depreciation and amortization), $Y_t - WP_t - \delta K_{t-1}$, is alternatively named $EBIT$ (Earnings Before Interest and Taxes). The operating cash inflow here must be exactly equal to the sum of the financial cash outflow and the cash outflow due to investment, by virtue of the fact that all increases in cash holdings (working capital) are by definition counted as capital investment.

Operating profit, $EBIT$, accounting profits before interest (and taxes), is defined by Π_t so that

$$(5) \qquad \Pi_t = Y_t - WP_t - \delta K_{t-1} \,.$$

And Net Income (NI_t) equals the accounting profits after debt payments (and taxes):

(6) $NI_t = Y_t - WP_t - \delta K_{t-1} - RP_t$.

The Free Cash Flow (CF_t) equals the operating cash flows after capital investment expenditures, which equals the cash outflows due to finance (because sources and uses of funds must be equal):

(7) $CF_t = Y_t - WP_t - I_t = D_t + RP_t - (NB_t^E + NB_t^D)$.

The book value of the firm's liabilities and net worth, B_t, accumulates from operating profit and via new issues of debt and equity, and diminishes with payouts to debt and equity holders. Thus,

(8) $B_t = B_{t-1} + \Pi_t - CF_t$.

Since the firm's capital stock evolves as

(9) $K_t = (1 - \delta) K_{t-1} + I_t$,

substituting equation (5) and the first part of equation (7) into (8), and given that $K_0 = B_0$, yields:

(10) $K_t = B_t$, for all t .

Conform the balance sheet provided in Table 1, the capital stock K_t can thus equally be viewed as either the book value of the firm's liabilities plus net worth or as the firm's total assets.

Table 1. Financial Accounting and Valuation

Income Statement	**Cash Flow Statement**
Revenues	*Sources of Funds*
Y_t (total income from operations)	$Y_t - WP_t$ (net cash flows from operations)
	$NB_t^E + NB_t^D$ (net cash from new finance)
Expenses	*Uses of Funds*
WP_t (wage and salary payments)	I_t (investment expenditures)
δK_{t-1} (depreciation allowances)	RP_t (interest to current debt holders)
RP_t (interest expenses equal to $r^f B_{t-1}^D$)	D_t (dividends to current equity holders)
Net Income = Revenues - Expenses	*Free Cash Flow*
$NI_t = Y_t - WP_t - \delta K_{t-1} - RP_t$	$CF_t = Y_t - WP_t - I_t = D_t + RP_t - (NB_t^E + NB_t^D)$.

Balance Sheet (*Book Values*)	**Balance Sheet (*Market Values*)**
Assets	*Assets*
$(1-\delta)K_{t-1}$ (remaining capital) +	K_t (current capital stock) +
I_t (gross investment) =	GO_t (present value of growth options) =
K_t (current capital stock)	P_t (market value of the firm)
Liabilities and Net Worth	*Liabilities and Net Worth*
B_{t-1}^D(existing debt) +	$V_{t-1}^D(1+r^f)$ (existing debt plus interest)
NB_t^D(new debt) =	$-$ RP_t (paid interest) + NB_t^D(new debt) =
B_t^D (current book value of debt) +	V_t^D (current value of debt) +
B_{t-1}^E (existing equity) $+NB_t^E$ (new equity)	$V_{t-1}^E(1+r_t^E)$(existing equity plus return)
+ $NI_t - D_t$ (retained earnings) =	$-D_t$ (dividends to equity holders) =
B_t^E (current book value of equity)	V_t^E (current intrinsic value of equity)
B_t (book value of the firm)	V_t (intrinsic value of the firm)

To relate *book value* to *market value*, define *economic profit* Π_t^* as

$$\text{(11)} \qquad \Pi_t^* = \Pi_t - \kappa_{t-1} K_{t-1} = Y_t - WP_t - (\delta + \kappa_{t-1}) K_{t-1} \ ,$$

equal to operating income minus the opportunity cost of providing and using the capital. The inclusive cost of capital represents a Jorgensonian "user cost" (Jorgenson, 1963) that includes depreciation, δ, as well as the financial gains foregone by not investing the capital in an alternative venture of similar risk, the required return κ_{t-1}). Then it follows from equations (9) and (11), and the first part of equation (7) together with equation (2), that

$$\text{(12)} \qquad V_t = K_t + \frac{E_t \Pi_{t+1}^*}{(1+\kappa_t)} + \frac{E_t \Pi_{t+2}^*}{(1+\kappa_t)(1+\kappa_{t+1})} + \ldots$$

So the value of the firm is equal to the book value of its assets, the capital stock, plus the present value of economic profits. If market efficiency prevails, so $P_t = V_t$, then equation (12) implies that market value may differ from book value if the present value of economic profits is not zero. If the balance sheet were marked to market then the capital stock would need to be augmented by a "goodwill" term generally referred to as the present value of *growth options*, GO_t :

$$\text{(13)} \qquad P_t = K_t + GO_t , \qquad GO_t \equiv \frac{E_t \Pi_{t+1}^*}{(1+\kappa_t)} + \frac{E_t \Pi_{t+2}^*}{(1+\kappa_t)(1+\kappa_{t+1})} + \ldots$$

The term growth options is a bit of a misnomer in the present context in that growth is not necessary for economic profits to be positive, and because the value of the "options" may actually be negative in cases where the capital stock "in place" has a *liquidation value* L_t lower than its book value. (Of course, this raises the question whether the opportunity cost is still $\kappa_t K_t$ and whether previous accounting adjustments should not have lowered the book value below its replacement cost K_t). Note that the growth options are zero if the *required return* is equal at all times to the *return on capital*, $\kappa_{t+j} = \Pi_{t+j+1}/K_{t+j}$ for all j. However, there is no obvious equilibrium requirement that would impose this equality, other than in an extremely competitive industry setting.

At times it is useful to distinguish the different types of capital assets, especially when we discuss asymmetric information and liquidity issues. We distinguish working capital WK_t (including here financial assets as well as cash equivalents), tangible capital TK_t (machines and buildings) and intangible capital IK_t (research in progress, copyrights, goodwill, etc.) Intangible capital is the problem category in the sense of causing asymmetric information issues – compared to outside investors, the insiders are likely to know more about the value of intangibles, and insiders may even be necessary labor inputs required to unlock the value of intangible assets – and in the sense of causing the associated liquidity issues – it may be difficult to sell intangible assets and they are unlikely to be useful as collateral. Hence, we may think of liquidation value relative to the value of capital in place as follows:

$$(14) \qquad L_t = K_t - IK_t \qquad (K_t \equiv WK_t + TK_t + IK_t).$$

In summary some of the key starting points in relating the various valuation concepts are as follows:

RESULT 5 (BASIC VALUATION IDENTITIES). *The intrinsic value of the firm, V_t (in equation 1) can equivalently be expressed in discounted cash flow form (equation 2), in certainty equivalent discounted cash flow form (equation 3), or in residual form as capital plus discounted economic profits (equation 12). The book value of the firm B_t equals the capital stock, $B_t = K_t$ (equation 10).*

(c) Distribution of Value among Financial Claimants

In an environment of perfect markets – no transactions costs, no taxes, symmetric information, unlimited short selling, perfect competition – basic insights may be obtained for valuing a firm based on the intrinsic value of equity and the intrinsic value of debt, and for comparing the required returns on equity and debt to the required return on the firm's assets as a whole.

The principle of "value additivity" provides Proposition I of Modigliani and Miller (1958):

$$(15) \qquad V_t = V_t^E + V_t^D .$$

The value of the firm's assets V_t is equal to the value of the firm's equity V_t^E plus the value of the firm's debt V_t^D. Thus, capital structure changes (such as issuing additional stock to pay off part of the debt) that do not affect the real operation of the firm have no impact on the value of the firm.

RESULT 6 (MODIGLIANI-MILLER PROPOSITION I). *In a perfect market, the intrinsic value of a firm is not affected by its capital structure as reflected in equation (15).*

A direct consequence of Result 6 is that we do not need to worry about diluting earnings when we issue new shares of equity because the overall market return on the firm's equity is identical to the returns of existing shareholders. To see this, we separate payments to shareholders into a dividend and new issue component: $D_{t+1} - NB_t^E = d_{t+1} n_t - (n_{t+1} - n_t) p_{t+1}^E$, in which lowercase symbols indicate values per share and n_t is the number of shares outstanding at time t. The equation assumes that new shares are issued at the appropriate market price. Using this equation to find the return on market equity gives: $1 + r_{t+1}^E = \{p_{t+1}^E n_{t+1} + [d_{t+1} n_t - (n_{t+1} - n_t) p_{t+1}^E]\}/p_t^E n_t = (p_{t+1}^E + d_{t+1})/p_t^E$. It follows that we do not need to distinguish between individual shareholder returns and the market returns of equity.

The intrinsic value of the firm, V_t , is given in equation (1). We can similarly write for the intrinsic value of the firm's debt, V_t^D:

$$(16) \qquad V_t^D = \frac{E_t(RP_{t+1} - NB_t^D + V_{t+1}^D)}{1 + \kappa_t^D} ,$$

with κ_t^D the required return on the firm's debt. Similarly, for the intrinsic value of the firm's equity:

$$(17) \qquad V_t^E = \frac{E_t(D_{t+1} - NB_t^E + V_{t+1}^E)}{1 + \kappa_t^E}$$

with κ_t^E the required return on the firm's equity. Combining equations (1), (16), and (17), and using the definition in equation (7), $CF_t = D_t + RP_t - (NB_t^E + NB_t^D)$, provides a version of Modigliani-Miller Proposition II:

$$(18) \qquad \kappa_t^E = \kappa_t + \lambda_t(\kappa_t - \kappa_t^D) , \qquad \lambda_t \equiv V_t^D/V_t^E .$$

RESULT 7 (MODIGLIANI-MILLER PROPOSITION II). *In a perfect market, the required return on the firm's assets is a value-weighted average of the required returns on the firm's equity and the firm's debt, as given in equation (18).*

Equation (18) states that the required return on market equity exceeds the required return on the firm's market assets by the ***leverage ratio*** $\lambda_t \equiv V_t^D/V_t^E$ times the difference between the required returns on equity and debt. Thus, for given asset risk, a more highly leveraged firm should have a higher required return on its equity and, given market efficiency, have a higher expected return.

The above identities also prove a proposition from Modigliani and Miller (1963) about the irrelevance of dividend policy. For a *given path of investment choices*, I_t for all t, subsitute equation (7), $CF_t = Y_t - WP_t - I_t$, into equation (1). Because the path of investment determines the capital stock at each time, and therefore operating income $Y_t - WP_t$, the timing of dividend payments D_t or issue of new equity NB_t^E has no impact on the value of the firm. With $CF_t = D_t + RP_t - (NB_t^E + NB_t^D)$ fixed, the timing of dividends and new equity only affects the timing of interest payments to debt holders and new debt. More precisely, higher dividends lower net debt payments or raise net new stock issues. Dividend issues may affect the capital structure of the firm but has no effect on its intrinsic value.

RESULT 8 (MODIGLIANI-MILLER PROPOSITION III). *In a perfect market, the timing of dividend payments has no impact on the intrinsic value of the firm.*

The result must hold because, in a perfect market, investors can on their own offset any reduction in dividend payments from the firm by selling enough of their equity to compensate for the reduction in cash dividends.

The same principles used to determine the various value indicators of the firm can also be used to determine the value indicators of equity. We have implicitly worked with a *clean surplus accounting* system according to which all additions to book value run through the income account. The analog of equation (8) for equity states, as in Table 1, that aside from net new issue of stock, the difference between net income and payments to equity holders contribute directly to the book value of equity:

$$(19) \qquad B_{t+1}^E = B_t^E + NB_t^E + (NI_{t+1} - D_{t+1}) \ .$$

The book value of the firm's equity, B_t^E, equals the book value of the previous period plus net new issue of equity and retained earnings. Retained earnings are given as the difference between net income, NI_t, and dividends, D_t. Net income represent the firm's accounting *earnings* equal to revenue net of labor costs, depreciation allowances, and interest payments on existing debt.

Substituting equation (19) into equation (17) yields:

$$(20) \qquad V_t^E = B_t^E + \frac{E_t(NI_{t+1} - \kappa_t^E B_t^E)}{(1+\kappa_t^E)} + \frac{E_t(NI_{t+2} - \kappa_{t+1}^E B_{t+1}^E)}{(1+\kappa_t^E)(1+\kappa_{t+1}^E)} + \ldots$$

The intrinsic value differs from the book value by the present value of *residual income* – the difference between net operating income and the opportunity cost of using the financial capital embedded in the book value of equity. Ohlson (1995) employs equation (20) together with the assumptions that residual income follows a specific ARMA process (with mean zero) and that the required return is constant, to calculate intrinsic value. See Lo and Lys (2000) for an insightful discussion.

Assuming an efficient market, $P_t^E = V_t^E$, it is worth noting that the expected return is still always equal to the required return, no matter how much intrinsic value differs from book value. For instance, given identical values of the required return, *value stocks* (those for which market value is below book value, $P_t^E < B_t^E$) and *growth stocks* (those for which market value is above book value, $P_t^E > B_t^E$) have the same expected returns.

Given that interest payments on the debt are guaranteed, as we assume, we know that $RP_{t+1} = r^f V_t^D$ and it is easy to confirm from Table 1 that the book value and intrinsic value of debt are always equal:

$$(21) \qquad V_t^D = B_t^D .$$

With this equality, comparing equation (20) to equation (12) and using Value Additivity (both in theory and accounting practice) implies that the growth options for equity equal the growth options for the firm as a whole:

$$(22) \qquad GO_t \equiv \frac{E_t \Pi^*_{t+1}}{(1+\kappa_t)} + \frac{E_t \Pi^*_{t+2}}{(1+\kappa_t)(1+\kappa_{t+1})} + \ldots = \frac{E_t(NI_{t+1} - \kappa_t^E B_t^E)}{(1+\kappa_t^E)} + \frac{E_t(NI_{t+2} - \kappa_{t+1}^E B_{t+1}^E)}{(1+\kappa_t^E)(1+\kappa_{t+1}^E)} + \ldots$$

So the present value of economic profits equals the present value of residual income. Note that residual income and economic profit are generally not identical in each period.[5]

[5] Using the fact that $GO_t(1+\kappa_t) - E_t\Pi^*_{t+1} = E_t GO_{t+1}$, by the definition of the growth options of the firm in equation (12), and a similar expression for the growth options of equity based on equation (21), it is straightforward to find that $sgn\left[\frac{E_t\Pi^*_{t+1}}{(1+\kappa_t)} - \frac{E_t(NI_{t+1} - \kappa_t^E B_t^E)}{(1+\kappa_t^E)}\right] = sgn\left[(\kappa_t - \kappa_t^E) E_t(GO_{t+1})\right]$. The difference between residual income and economic profit depends on $\kappa_t^E - \kappa_t - \lambda_t^B(\kappa_t - \kappa_t^D)$, $\lambda_t^B \equiv B_t^D / B_t^E$ for each individual period, which is negative by comparison with equation (17) (since $\lambda_t^B > \lambda_t$ if growth options are positive). In contrast, if growth options are expected to be positive, $E_t(GO_{t+1}) > 0$, and the firm is levered (so that $\kappa_t < \kappa_t^E$), then the *discounted value* of next period's residual income is expected to exceed the discounted value of next period's economic profit income.

(d) Common Errors in Valuation

While the valuation approaches in general are straightforward, they are often confounded in practice. Fernandez and Bilan (2007) point out a series of common mistakes in the application of valuation methods in corporate practice, a few of which we highlight here.

1. *Nominal versus Real*. As we saw in section 2(e) above, it is fine to value assets by discounting nominal future expected cash flows, as long as the discount rate is also nominal. A typical error is to account for future inflation by calculating future cash flows with a real growth rate but leaving discount rates in nominal terms. The *Fed model* in fact makes exactly this conceptual error as we discuss in section 8(a) below.

2. *Predictable Returns*. It is a common misperception to believe that market efficiency must mean that we cannot, on average, expect positive growth in stock prices. Balvers, Cosimano, and McDonald (1990) point out, however, that aggregate stock market returns depend on forecastable macroeconomic conditions which leads to predictable increases in stock market index prices even when the market is perfectly efficient. In the context of the Gordon Growth Model, for instance, when market efficiency implies that price equals present value, the required return κ equals the dividen yield, equal to $\kappa - g$ from equation (4), plus the capital gain, which must therefore be equal to g. It follows that the stock price is expected to grow at rate g. There are two reasons for this predictable growth trend. First, the nominal component due to inflation. Second, real growth due to a trend in overall (macroeconomic) growth. The latter is related also to earnings retention: from the dividend-neutrality result, Result 8 above, the mean return is not affected by dividend payout. So, if the dividend yield falls below $\kappa - g$ due to more earnings being retained, then the stock price grows faster than rate g. Note that issuing additional shares (earnings dilution) has the same effect as earnings retention, causing faster growth in the stock's price. Consider the Dow Jones Industrial Index (DJIA) as an example. It passed the 10,000 level for the first time in 1999. In 2010 it dipped below 10,000 again. This is definitely not the outcome that should have been expected back in 1999. The presence of inflation alone, at an average rate of 2.6% over this period, on average ought to have increased the DJIA to 13,260. In addition, retention of earnings should have led to further increases in the DJIA, on the average. In short, valuation of a company, esp. one with substantial holdings of financial assets such as a financial corporation, must build in positive expected capital gains.

3. *Adjusting for Leverage.* A company with a higher debt-to-equity ratio is leveraging

operating income for its shareholders. This fact must be considered in calculating the value of equity. But, it is crucial to also consider the appropriate required return in finding the present value of dividends. It is tempting to take the required return of, say, a close competitor as the proper discount rate. However, the competitor may be similar in terms of its operations but have a different capital structure. The correct approach would be to use the competitor's required return on market assets and adjust it for leverage according to Result 7, equation 18.

4. *Default Risk*. As discussed in section 3(a), it is important to distinguish between one-sided and two-sided risk. If we value a corporate bond with substantial default risk, the correction for default risk should be made by adjusting the expected values of future promised payments. Once that is accomplished no further adjustment may be necessary; in particular there may be no need to include a risk premium in the discount rate. A risk premium is only necessary for priced systematic risk. In this case it implies that the discount rate needs no risk premium if the default risk is idiosyncratic, i.e., not correlated with the market, systematic. For the proper risk-free discount rate, see the next item.

5. *Duration of Cash Flows*. The discount rate depends non-linearly on the timing of when payments mature. The issue is clearest when discounting promised future cash flows, as in valuing a corporate bond. On first impulse one may like to use a constant risk-free rate (plus maybe a constant risk premium to account for any systematic default risk). However, payments that are postponed longer tend to have higher average discount rates. This is consistent with longer-term government bonds usually having higher average returns – that the yield curve on average has a positive slope. Thus, the right way to discount future coupon payments is to discount each by the government bond rate of similar maturity (plus risk premium if necessary).

6. *Earnings versus Dividends.* Because dividend policies are difficult to forecast it is tempting to discount earnings instead of dividends as the basis for equity valuation. The problem is that earnings do not represent the cash flows going directly to the equity holders. Any retained earnings end up with the equity holders eventually but with a lag, which due to the time value of money of course is not the same. Retained earnings do, of course, finance investment leading to higher future earnings and dividends but this is already taken into account in obtaining the projections for future dividends. The proper way to deal with earnings in valuation is discussed in the next section.

5. Earnings and Dividends in Valuation

Valuation often relies on ratios such as the *dividend yield* (the dividend-to-price ratio), the *PE ratio* (the price-to-earnings ratio), and the *book-to-market ratio* (book value of equity divided by market value of equity) to provide a quick approximation of whether an asset is priced reasonably and to see what future returns may be expected. We make some further simplifying assumptions to obtain a perspective on what these ratios may or may not tell us.

(a) Earnings and the Gordon Growth Model

First consider that earnings in this context are identical to what we earlier referred to as net income: $\mathit{Earnings} \equiv NI$. In the Gordon Growth Model the assumptions are that both the required return on market equity and the growth rate of dividends are constant over time. To involve earnings a further assumption is made that the *payout ratio* is constant. The payout ratio is the fraction b of net income that is paid out in the form of dividends: $b = D_t / NI_t \leq 1$. Similarly, the fraction $1 - b$ is the *retention ratio*: retained earnings divided by net income.

If we leave the growth rate of dividends constant and apply equation (4.4) to equity, then:

$$(1) \qquad V_t^E = \frac{E_t D_{t+1}}{\kappa^E - g} = \frac{b\, E_t(NI_{t+1})}{\kappa^E - g} .$$

In an efficient market we accordingly have,

$$(2) \qquad \frac{E_t D_{t+1}}{P_t^E} = \kappa^E - g \leq \frac{E_t(NI_{t+1})}{P_t^E} = \frac{\kappa^E - g}{b} .$$

Thus, we expect dividend yields to be smaller than earnings yields (or price-dividend ratios larger than price-earnings ratios) if payout ratios are below one. Since $E_t D_{t+1} = (1+g) E(D_t)$ and $E_t NI_{t+1} = (1+g) E(NI_t)$ both ratios should be larger by a, relatively small, factor $1+g$ because price is divided by expected *current,* not *future*, dividends or earnings. Note that we use *expected* current dividends or earnings because the capital stock and hence growth is not affected by *unexpected* earnings. In practice, to calculate price-dividend or price-earnings ratios price is often divided by an average of *trailing* dividends or earnings which averages out temporary earnings shocks.

A problem with this specification based on Gordon (1959) is that it is not clear under which circumstances the assumed constancy of the growth rate g and the payout ratio b are mutually compatible and, for instance, runs afoul of the second Modigliani-Miller proposition (Result 7 above): if retained earnings are used for investment, growth is different than if retained earnings are used to reduce debt. The utilization of retained earnings affects the leverage ratio and, accordingly, the required return on market equity κ^E.

A more specific set of assumptions employed by Gordon (1959) posits, in addition, that (a) the *Return on Investment* ($ROI_{t+1} = NI_{t+1}/K_t$ if debt is zero) is constant and equal to the required return on market equity, κ^E, and (b) there is no debt financing.[6] Then the growth rate of the capital stock and, hence, earnings must be equal to the retention ratio times the ROI: $g = (1-b)\kappa^E$. In this case it follows that

$$(3) \qquad \frac{E_t D_{t+1}}{P_t^E} = b\,\kappa^E \leq \frac{E_t(NI_{t+1})}{P_t^E} = \kappa^E \, .$$

So for the $ROI = \kappa^E$ and the *no debt* case the dividend yield remains generally smaller than the earnings yield, as for the case of constant payout ratio in equation (2). Note that the required return κ^E here is *nominal*. The issue does not arise when we interpret the dividend yield $E_t D_{t+1}/P_t^E = \kappa^E - g$ from equation (2) rather than the earnings yield, because the dividend yield is related to the required return net of the growth rate, so the result is the same whether we use the nominal or the real required return and growth rate. Since we do not subtract a growth rate in interpreting the earnings yield in equation (3) it is important to be clear on whether the required return is real or nominal. The key is that the expected future earnings should include an inflation premium to compensate the investor for the loss of purchasing power of the initial investment that must be recovered.

Below we take an alternative look at these ratios based on the more attractive model used previously, which includes debt and allows the ROI to differ from the required return on market equity, but also avoids the conflicts with the Modigliani-Miller result and constant growth.

(b) Earnings and the Return on Investment

The key assumption in the present subsection is that expected operating profits (or EBIT) equal a constant fraction h of the capital stock in place:

$$(4) \qquad E_t \Pi_{t+1} = E_t(Y_{t+1} - WP_{t+1} - \delta K_t) = hK_t \, .$$

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[6] The *Return on Investment*, $ROI_{t+1} = \Pi_{t+1}/K_t$, and the *Return on Assets*, ROA_{t+1}, are equal given constant returns to scale. A third commonly used accounting measure of return is the *Return on Equity*: $ROE_{t+1} = NI_{t+1}/B_t^E$.

Growth occurs only when the capital stock grows. To generate constant growth of g, net investment must be equal to a fraction g of the capital stock:

$$(5) \qquad I_{t+1} - \delta K_t = gK_t.$$

From equations (4) and (5), expected cash flow available to the financial claimants of the firm (remember that $CF_t = D_t + RP_t = Y_t - WP_t - I_t$) accordingly equals

$$(6) \qquad E_t CF_{t+1} = (h - g) K_t .$$

Since the capital stock grows at rate g, the cash flows, from equation (6), are expected to grow at rate g as well. Under constant returns to scale, the cost of capital κ (the required return on the firm's assets) can reasonably be expected to be constant as well, as the capital stock accumulates. Hence, capitalizing future cash flows gives the value of the firm as

$$(7) \qquad V_t = \left(\frac{h - g}{\kappa - g} \right) K_t .$$

In summary,

> **RESULT 9 (VALUATION RATIOS).** *Assume* constant *(a) capital growth* g, *(b) return on investment* h, *and (c) cost of capital* κ. *Then the* value of the firm V_t *is given by equation (7). The* value V_t *exceeds the* value of capital K_t *and the* market value of equity V_t^E *exceeds the* book value of equity B_t^E *if and only if* $h > \kappa$.

Note that with the maintained assumption of market value of debt equal to book value of debt, when the market value of assets exceeds the value of capital (the book value of assets) it is obvious that the market value of equity exceeds the book value of equity.

The firm has "growth options" ($V_t > K_t$) as long as the ROI (h) is above the cost of capital (κ). How reasonable is this? Consider that any investment project is accepted if its *Net Present Value* (NPV) is positive. Hence, the *marginal* investment project has NPV = 0. For this marginal project we have $h_{marg} = \kappa$. The infra-marginal projects will generally have ROI larger than the cost of capital. If we think of the capital stock as consting of a portfolio of projects, each growing in the same proportion, then it is not unreasonable for $h > \kappa$, reflecting the fact that the average project has positive NPV.

The separate determination of the value of the firm's equity and debt is more complex. Setting the payout ratio constant to provide an expression for the value of equity poses problems since it generally implies that the firm's leverage ratio will change with time, causing the required return on market equity to change (Result 7). Instead, it is simpler to assume that the firm adjusts its financing over time to keep the leverage ratio constant $\lambda_t = \lambda$ (supposedly at a level that provides the optimal capital structure). Obviously then the value of equity (or of debt) is a constant fraction of the value of the firm and can be inferred from equation (7). However, finding the price-earnings ratio or its inverse the earnings yield is harder, as is derived next.

$$(8) \qquad V_t^D / V_t^E = \lambda \,, \quad \text{for all } t.$$

Since earnings consist of net operating income minus interest payments we have $E_t NI_{t+1} = h K_t - k^D B_t^D$. By definition of constant leverage and because book value of debt equals market value of debt we have that $k^D B_t^D = r^f \lambda \, V_t^E$. From equation (7) and equation (4.17) we derive the earnings yield:

$$(9) \qquad \frac{E_t(NI_{t+1})}{P_t^E} = \kappa^E - g\left(\frac{(1+\lambda)(h-\kappa)}{h-g}\right).$$

An alternative value indicator is the book-to-market ratio. To find it we simply take the book value of equity as the capital stock minus the market value of debt; then use the constant leverage ratio to relate the market values of debt and equity to the value of the firm, and employ equation (7) to generate

$$(10) \qquad \frac{B_t^E}{P_t^E} = 1 - \left(\frac{(1+\lambda)(h-\kappa)}{h-g}\right).$$

Clearly there is a simple relation between the earnings yield and the book-to-market ratio:

$$(11) \qquad E_t(NI_{t+1})/P_t^E = (\kappa^E - g) + g\,(B_t^E/P_t^E).$$

A linear regression between firm book-to-market ratios and earnings yields over time should identify both the firm's anticipated growth rate and its required return on market equity. Note from equation

(2) that we can substitute the dividend yield for the first term on the right-hand side on equation (11). Thus the difference between earnings and dividend yield divided by the book-to-market ratio should provide the anticipated growth rate of the firm. This simply results from the fact that retained earnings grow in proportion to the book value of equity according to equation (4.18).

We have clear results for valuation and valuation ratios under a set of specific assumptions:

RESULT 10 (EQUITY VALUATION RATIOS). *Assume* constant *(a) capital growth* g *(b) return on investment* h *(c) costs of capital* κ *and debt* r^f *and (d) leverage ratio* λ. *Then the* earnings yield *and* book-to-market *ratios are as given in equations (9) and (11).* Earnings yield *exceeds* dividend yield *if and only if* $g > 0$.

Windfall gains and growth

The model assumes risky earnings that must be in proportion to the capital stock to warrant a constant cost of capital (required return on the assets). In effect:

$$\text{(12)} \qquad \Pi_{t+1} = h_{t+1} K_t \,, \quad h_{t+1} = h + \omega_{t+1} \,,$$

where ω_t is i.i.d. (independently and identically distributed) with mean of zero: the shock to the realized ROI is assumed to be white noise and does not persist over time. It is easy to confirm that the windfall gain directly benefits the stockholders of the firm who receive leveraged gains: $r^E_{t+1} = \kappa^E + (1+\lambda)\,\omega_{t+1}$. These one-off gains should not affect the growth plans of the firm. Given constant investment policy and leverage ratio, the only possible action is for the firm to use the windfall gain to temporarily raise dividends (or buy back shares). It is easy to confirm from Table 1 that then the book value of equity and the market value of equity both remain unaffected by the temporary gain. The result agrees with the general Modigliani-Miller implication that valuation does not depend on the state of finances.

6. VALUATION WITH STOCHASTIC DISCOUNT FACTORS

Under the assumption of perfect (but not necessarily complete) capital markets it is possible to derive a "stochastic discount factor" that prices every asset. The key aspects of the perfectness assumption here are that portfolios of assets can be formed freely without transactions cost and without short-selling constraints, and that no arbitrage opportunities exist. Under these assumptions there always exists a positive stochastic discount factor that serves to discount any cash flow. Assume for simplicity that market prices equal intrinsic valuations. Then, for any asset (or project) i with one-time cash flow X^i_{t+1}, value is given as:

$$(1) \qquad V^i_t = E_t(m_{t+1}X^i_{t+1}) \ ,$$

where m_{t+1} is the *stochastic discount factor* for period $t+1$. Compared to a standard discount factor: (a) it is not a deterministic constant, its outcome is stochastic, and (b) it does not differ by asset, there is no superscript i. If an asset with known cash flow exists, this risk free asset has a known return of r^f_t and it follows that the expected value of the stochastic discount factor is equal to the standard risk-free discount factor:

$$(2) \qquad E_t(m_{t+1}) = \frac{1}{1 + r^f_t} \ ,$$

as can be inferred from equation (1) for a known cash flow divided by its price.

Using the formula of covariance (see equation C5 in the Appendix) together with equation (2) in equation (1):

$$(3) \qquad V^i_t = \frac{E_t(X^i_{t+1}) + Cov_t(m_{t+1}/E_t m_{t+1}, X^i_{t+1})}{1 + r^f_t} \ .$$

Equation (3) represents the certainty-equivalent form that is a generalized version of equation (3.9), which holds only in a specific CAPM context (it turns out that equation 3.7 can be derived from equation 3 because the stochastic discount factor in the CAPM is a deterministic linear function of the market return). Alternatively, defining the return as $r^i_{t+1} \equiv X^i_{t+1} / V^i_t$:

$$(4) \qquad V^i_t = \frac{E_t(X^i_{t+1})}{1 + \kappa^i_t} \ , \quad \kappa^i_t = r^f_t - Cov_t(m_{t+1}/E_t m_{t+1} \, , r^i_{t+1}).$$

Equation (4) provides the standard valuation expression where expected cash flows are discounted by the appropriate required return for this particular asset. Even though the same stochastic discount factor prices all assets, once we transform to the deterministic discount formulation the discount factor becomes asset specific. Note that $-\ Cov_t(m_{t+1}/E_t m_{t+1} \, , r^i_{t+1})$ in the required return can be interpreted as a *risk* premium because the covariance is typically negative; it can moreover be interpreted as a *systematic risk* premium, even outside the CAPM context, because the covariant term $m_{t+1}/E_t m_{t+1}$ is a system wide variable, identical for all assets.

To consider intertemporal valuation issues, we may write $X^i_{t+1} = D^i_{t+1} + V^i_{t+1}$, yielding an expression similar to equation (4.1):

$$(5) \qquad V^i_t = \frac{E_t(D^i_{t+1} + V^i_{t+1})}{1 + \kappa^i_t}\ , \quad \kappa^i_t = r^f_t - Cov_t(m_{t+1}/E_t m_{t+1}, r^i_{t+1}).$$

If the required returns κ^i_{t+j} for all j are deterministic then equation (5) may be solved forward to yield an equation like (4.2). Solving forward directly from equation (1), on the other hand, when $X^i_{t+1} = D^i_{t+1} + V^i_{t+1}$ gives:

$$(6) \qquad V^i_t = E_t(m_{t+1} D^i_{t+1}) + E_t(M^{t+2}_t D^i_{t+2}) + E_t(M^{t+3}_t D^i_{t+3}) + \dots\ ,$$

where $M^{t+j}_t \equiv \prod_{i=1}^{j} m_{t+i}$. As in equation (3) we can derive a certainty-equivalent expression. If a riskless bond exists for each maturity j with yield to maturity r^{t+j}_t then $(1 + r^{t+j}_t)^j = 1/E_t(M^{t+j}_t)$. Thus:

$$(7) \qquad V^i_t = \frac{E_t(D^i_{t+1}) + Cov_t(m_{t+1}/E_t m_{t+1}\ , D^i_{t+1})}{1 + r^{t+1}_t} + \frac{E_t(D^i_{t+2}) + Cov_t(M^{t+2}_t/E_t M^{t+2}_t\ , D^i_{t+2})}{(1 + r^{t+2}_t)^2} + \dots$$

The risk-adjusted cash flows are discounted by the appropriate yield to maturity for that period. If the yield curve is upward sloping then the discount rate increases with the horizon. Equation (7) illustrates the common valuation error issue from section 5(d) regarding the duration of payments: the proper risk free rate varies with the time of payment.

In a specific general equilibrium context we can often identify the stochastic discount factor as the marginal rate of intertemporal substitution. For a standard separable utility specification then $m_{t+j} = \beta u'(c_{t+j})/u'(c_{t+j-1})$ so that $M^{t+j}_t = \beta^j u'(c_{t+j})/u'(c_t)$. Equation (7) becomes accordingly:

$$(8) \quad V^i_t = \frac{E_t(D^i_{t+1}) + Cov_t[u'(c_{t+1})/E_t u'(c_{t+1}), D^i_{t+1}]}{1 + r^{t+1}_t} + \frac{E_t(D^i_{t+2}) + Cov_t[u'(c_{t+2})/E_t u'(c_{t+2}), D^i_{t+2}]}{(1 + r^{t+2}_t)^2} + \dots$$

A full comparison between valuation with the stochastic discount factor formulation of equation (6) and valuation with deterministic discount rates as in equation (4.2) requires converting the certainty equivalence expression of equation (7). The yields to maturity, in the denominators of equation (7), can be shown to be related to implied forward rates (the future one-period risk free rates that can be secured at the present time by buying and shorting risk free bonds of maturities one period apart):

$$(9) \qquad (1 + r_t^{t+j})^j \equiv \prod_{s=1}^{j} (1 + f_{t,t+s}) \ ,$$

where $r_t^{t+1} \equiv r_t^f \equiv f_{t,t+1}$ and $1 + f_{t,t+j} \equiv E_t(M_t^{t+j-1})/E_t(M_t^{t+j}) = 1/E_t(m_{t+j})$. (The last equality holds by a standard arbitrage argument). The deterministic discount rates in the discounted expected dividends expression in equation (10) are chosen so that valuation is identical to that in the stochastic discount expression (6). We could obtain the right expression by solving forward the first-order difference equation (5) but only if, conditional on information at the current time t, future discount rates κ_{t+j}^i are independent of future dividends D_{t+j+1}^i, which is not generally the case. Instead choose alternative discount rates $1 + \kappa_{t,t+j}^i = [E_t(D_{t+j}^i + V_{t+j}^i)]/E_t(V_{t+j-1}^i)$.[7] Then:

$$(10) \qquad V_t^i = \frac{E_t D_{t+1}^i}{(1 + \kappa_{t+1}^i)} + \frac{E_t D_{t+2}^i}{(1 + \kappa_{t+1}^i)(1 + \kappa_{t,t+2}^i)} + \frac{E_t D_{t+3}^i}{(1 + \kappa_{t+1}^i)(1 + \kappa_{t,t+2}^i)(1 + \kappa_{t,t+3}^i)} + \ldots,$$

$$\kappa_{t,t+j}^i = f_{t,t+j} - Cov_t(m_{t+j}/E_t m_{t+j}, \tilde{\kappa}_{t,t+j}^i), \quad \tilde{\kappa}_{t,t+j}^i = (D_{t+j}^i + V_{t+j}^i - V_{t+j-1}^i)/E_t(V_{t+j-1}^i),$$

where $\kappa_{t,t+j}^i = E_t(\tilde{\kappa}_{t,t+j}^i)$ is derived from $V_{t+j-1}^i = E_{t+j-1}[(m_{t+j}(D_{t+j}^i + V_{t+j}^i)]$: taking current expectations on both sides and using the definition $\tilde{\kappa}_{t,t+j}^i = (D_{t+j}^i + V_{t+j}^i - V_{t+j-1}^i)/E_t(V_{t+j-1}^i)$ implies that $1 = E_t(m_{t+j}\tilde{\kappa}_{t,t+j}^i)$ which yields $\kappa_{t,t+j}^i = f_{t,t+j} - Cov_t(m_{t+j}/E_t m_{t+j}, \tilde{\kappa}_{t,t+j}^i)$ given that $1 + f_{t,t+j} = 1/E_t(m_{t+j})$.

The required return $\kappa_{t,t+j}^i$ includes two risk premia: one related to the liquidity risk from receiving cash flows later in time $f_{t,t+j} - E_t(r_{t+j}^f)$, reflected in the difference between the implied forward rate and the expected future spot rate and fixed across assets, and one related to an asset-specific covariance risk, $-Cov_t(m_{t+j}/E_t m_{t+j}, \tilde{\kappa}_{t,t+j}^i)$. Abel (1999) provides a specific general

[7] In equation (10), $\kappa_{t,t+j}^i \neq \kappa_{t+j}^i = E_t r_{t+j}^i$ so that result 4, by which required return equals expected result, does not apply. Even if there is no covariance risk, we would have $\kappa_{t,t+j}^i = f_{t,t+j} \neq E_t r_{t+j}^f$, where the implicit forward rate is not equal to the expected spot risk free rate unless the expectations hypothesis of the term structure holds.

equilibrium treatment of these separate components of the risk premium; Ang and Liu (2004) provide specific formulas for obtaining the intrinsic value when required returns are determined in the context of a conditional CAPM with time-varying betas and risk premia.

7. APPROACHES FOR DERIVING EXPECTED RETURNS

The general approaches discussed in the introduction for valuing assets—intrinsic value, comparative value, and replacement value—can with some modifications each be applied to derive expected returns. We discuss these approaches in turn and provide a specific example.

(a) Intrinsic Value

Lee, Myers, and Swaminathan (1999) assume that market value and intrinsic value (fundamental value) are cointegrated processes. In this case an error correction model can be estimated that provides an estimate for the speed of reversion of market value to intrinsic value, which is a crucial ingredient for determining expected returns when the market and intrinsic values are not identical. For simplicity we assume here an efficient markets approach so that market value equals intrinsic value at all times. Based on equation (4.17) we can then use accounting information and the market value of equity to back out values for the required rates of return. This becomes difficult when the required rates of return vary over time so we assume them to be constant.

A simple example follows from the Gordon Growth Model (Gordon and Shapiro, 1956, Gordon, 1959, 1962). Estimate dividends for the upcoming period and a growth rate for expected dividends. The required rate of return can then be obtained from equation (5.1) since intrinsic value is observable as market value. Because the expected return must equal required return, the empirical prediction is that average future returns are equal to the inferred required return. Fama and French (2002) use this model to infer the required return on the S&P500 index. They find that, in contrast to the average equity premium of 8.28%, the current equity premium inferred from the Gordon Growth Model is around 4%. Their explanation is that unexpected decreases in the required return have led to capital gains exaggerating past returns. Balke and Wohar (2001) make a similar point using this intrinsic value approach. In section 8 we examine historical data on S&P500 index returns, earnings, and dividends to apply the intrinsic valuation approach

A more sophisticated version of the Gordon Growth Model is based on the Ohlson (1995)

Model. Starting from equation (4.20), Ohlson assumes (a) a constant required return, $\kappa_t^E = \kappa^E$ for all t, and (b) an ARMA process (with unconditional mean of zero) for realized residual income, $X_{t+1} = NI_{t+1} - \kappa^E B_t^E$. The ARMA process is specified as $X_{t+1} = \rho X_t + \nu_t + \varepsilon_{t+1}$ with $\nu_{t+1} = \gamma \nu_t + \eta_{t+1}$ and ε_t, η_t i.i.d. mean zero processes. Then it is easy to derive that

$$(1) \qquad V_t^E = B_t^E + \frac{\rho}{(\kappa^E - \rho)} X_t + \frac{\kappa^E}{(\kappa^E - \rho)(\kappa^E - \gamma)} \nu_t \, .$$

If we assume market efficiency so that $P_t^E = V_t^E$ the above equation yields κ^E. A technical problem is that estimation of γ, ρ requires knowledge of $X_{t+1} = NI_{t+1} - \kappa^E B_t^E$ which in turn requires a value for κ^E. But an iterative estimation procedure starting with a reasonable guess for κ^E should quickly converge. It is straightforward to show that if we ignore ν_t then the Ohlson Model becomes the Gordon Growth Model.

Market information together with accounting information for calculating intrinsic value allows an estimate of the required return. If market efficiency is imposed then average returns can be forecast structurally without requiring a specific model of risk. While such an approach seems a reasonable competitor for standard asset pricing models, it is not widely applied.

(b) Comparative Value

The standard approach for obtaining required returns is to compare the asset under consideration to other assets that are *similar* in important respects. If the assets to be compared are *equivalent* in terms of payoffs, this is called arbitrage. In other cases, differences in required returns occur and must be related to differences in risk. Approaches differ depending on how risk is defined and measured. Assets with the same "risk" should be assigned the same required return. In the arbitrage and risk class approaches, required asset returns are specified relative to that of, respectively, underlying assets (primary securities) or benchmark assets (such as the market in the case of the CAPM). Assets can also be compared to past versions of themselves.

Arbitrage

Absence of arbitrage opportunities in a perfect market implies that different portfolios yielding the same payoffs in all states must have the same price. This fact can be used to price any derivative asset in relation to the underlying primary asset. The Black-Scholes option pricing formula for instance stems from constructing a hedging portfolio consisting of one share of the underlying asset and an appropriate number of call options on this asset such that the payoff becomes riskless and must equal the gross return on a riskless asset times the initial investment.

Relative Risk Exposure

Most "asset pricing" models foremost yield expressions for required returns which, as an afterthought, can be converted to valuation expressions according to equation (4.2) if information on expected dividends is introduced. The required returns are derived relative to benchmark assets, such as a risk free asset and the market index, based on the asset's riskiness. Theories differ on what constitutes risk and which risk is priced in the market, but the general idea is that two assets with identical exposure to the relevant risk factors should have identical required returns, even if their payoffs are generally different in different states. The relevant priced risk is typically the *systematic* risk that concerns only the return variability that does not disappear in a well-diversified portfolio.

Past Returns

Another comparison is to consider the asset a good substitute for the past version of itself and to assume that the overall market valuation of risk is stable over time. In this case a required return can be obtained without an explicit model of risk. The simplest approach would be to calculate the average past returns going back a particular number of periods. A slightly more sophisticated method would be to calculate the past average risk premium and to add this to the known risk-free rate over the future period under consideration. One may also want to put less weight on older observations. A practical problem with the latter is that the weights or, similarly, how far back to go in calculating the average, are arbitrary. Nevertheless, for instance for capital budgeting purposes, this crude approach may be preferred over standard risk-based models, which are not particularly successful empirically. Goyal and Welch (2008) apply this type of approach to the U.S. market return. They find that the historical average of past market excess returns, as measured by the S&P500 index return over the T-Bill rate, is a better predictor of the future market return than any alternative model using standard forecast variables such as aggregate price-dividend ratios, book-to-market values, and S&P return variances.

(c) Replacement Value

Replacement value identifies the value of an asset as the cost of replicating and putting in place the asset's capital. In principle, replacement cost is an appropriate criterion for valuation in a variety of circumstances. If a firm holds an optimal level of inventories, for example, it makes sense to value these, not at their historical cost or what they sell for, but for what it costs to replace them. The same idea applies to a firm's total assets (the capital stock). If assets are valued at replacement cost in the books then the book value/capital stock of the firm represents the replacement value of the firm.

From equations (4.1) and (4.6) if we set $V_t = K_t$ we obtain $1 + \kappa_t = E_t(D_{t+1} + RP_{t+1} + K_{t+1})/K_t$ and, also derive $\kappa_t = E_t(Y_{t+1} - W_{t+1}N_{t+1} - I_{t+1} + \Delta K_{t+1})/K_t = [E_t(Y_{t+1} - W_{t+1}N_{t+1})/K_t] - \delta$ from equations (4.6) and (4.9). Assuming a constant returns to scale production function and optimal input choice so that $Y_{t+1} = W_{t+1}N_{t+1} + R^K_{t+1}K_t$ then $\kappa_t + \delta = E_t R^K_{t+1}$. The required return equals the expected user cost of capital minus the rate of depreciation of capital. To find an explicit expression, assume a Cobb-Douglas production function a with multiplicative productivity shock, which generates $\kappa_t + \delta = (1-\gamma)E_t(\theta_{t+1})K_t^{-\gamma}$. Thus, if markets price firms in this way, expected returns should equal required returns and depend on the marginal/average productivity of capital which depends positively on expected productivity and negatively on the capital stock. Note that expected returns are determined without reference to expected returns *on other assets* in this approach.

The investment-based asset pricing model of Cochrane (1991) and Restoy and Rockinger (1994) and the valuation approach of Hall (2001b) are based on the replacement value principle but are more sophisticated in that they allow for adjustment costs. If adjustment costs can be ignored their return equation becomes identical to the one derived here. The value of a firm is not generally equal to the capital stock if there are costs of adjustment in the capital stock. Tobin (1969) argues that physical investment is determined positively by the *q-ratio* – the ratio of the firm's market value to the replacement value of its capital stock (the book value), basically the inverse of the book-to-market ratio. Net investment is positive (negative) when $q > 1$ ($q < 1$) and zero when $q = 1$, so it follows that market value moves towards book value in the absence of exogenous shocks, but slowly.

8. APPLICATIONS AND EXERCISES

* **Application 1**: *Money Illusion and Valuation*

The mistake of discounting real cash flows by nominal required returns appears to be incredible, but it seems that the "Fed model", the leading practitioner model for equity valuation (in spite of the name, not the official model of the Federal Reserve System) makes the same mistake.

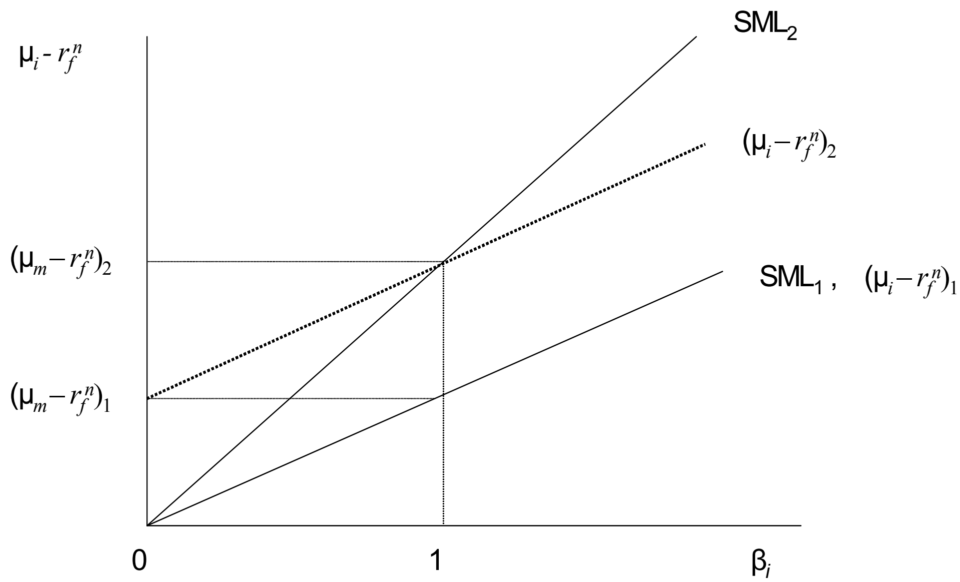

Figure 4
Stock Returns and Money Illusion
Anticipated inflation above the long-run average implies higher mean excess returns for all stocks according to the Modigliani-Cohn hypothesis. As this holds for the mean market excess return as well, stocks with betas below one have positive alphas and those with betas above one have negative alphas.

Cohen, Polk, and Vuolteenaho (2005) investigate the *Modigliani-Cohn hypothesis* – that stock prices are determined as if real cash flows are discounted by nominal required returns– by determining and examining the implications for cross sectional differences in stock returns.

Cohen, Polk, and Vuolteenaho assume that stock prices are set according to the Gordon Growth Model (constant dividend growth g) but by investors subject to the sort of money illusion conjectured by Modigliani and Cohn. Thus for some firm i

$$\text{(A1)} \qquad P_{it}^{E} = \frac{E_t D_{it+1}}{\kappa_i^E - \hat{g}_i} \,,$$

where κ_i^E is the nominal required return and $\hat{g}_i$ is the real growth rate of cash flows/dividends. Taking a flexible perspective on the Modigliani-Cohn hypothesis we may assume that the illusion is potentially not complete so that we have

$$\text{(A2)} \qquad \hat{g}_i = g_i - f(\pi) .$$

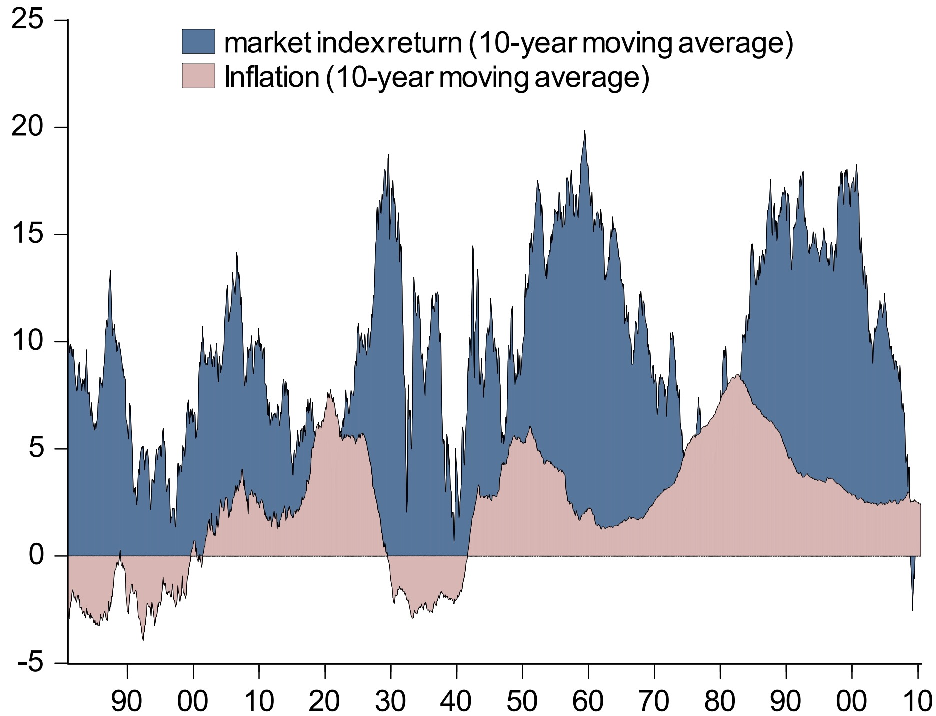

Figure 5
Historical Nominal Market Returns and Inflation

If money illusion is complete we would have $f(\pi) = \pi$. However, if money illusion is incomplete and growth estimates are based on long term averages $\bar{\pi}$ it may be more reasonable to set $f(\pi) = \gamma(\pi - \bar{\pi})$, with $0 < \gamma < 1$.

We may now calculate the expected return by adding expected dividend yield ($E_t D_{it+1} / P^E_{it} = \kappa^E_i - \hat{g}_i$, from equation A1) and capital gain $(E_t P_{it+1} - P_{it}) / P^E_{it} = g_i$, from equation A1 and using actual dividend growth):

$$\text{(A3)} \qquad \mu_i = \kappa^E_i + f(\pi),$$

which follows by adding dividend yield and capital gain and using equation (A2). Thus mean returns for any stock *i* exceed the required return on stock *i* by the same percent which depends positively on inflation. If we assume, as do Cohen, Polk, and Vuolteenaho (2005), that the required returns satisfy the Capital Asset Pricing Model then:

$$\text{(A4)} \quad \kappa_i^E - r_f^n = \beta_i(\kappa_m^E - r_f^n),$$

where r_f^n is the nominal risk free rate, β_i is the stock's "beta", its sensitivity to market risk, and κ_m^E is the required return on the market portfolio. Apply equation (A3) to both a generic stock *i* and the market portfolio *m* and substitute into equation (A4):

$$\text{(A5)} \quad \mu_i - r_f^n = \beta_i(\mu_m - r_f^n) + (1 - \beta_i)f(\pi).$$

The part of the expected return unexplained by the CAPM, the asset's "alpha" is given as $\alpha_i = (1 - \beta_i)f(\pi)$. Thus, if we test the CAPM at a time of high inflation we find that all returns are "too high" by the same percent, as illustrated in Figure 4. The reason is that anticipated high inflation causes high discount rates but leaves the anticipated cash flow growth rate (partly) unchanged. So prices are lower, raising the dividend yield and average stock returns realized in actuality. The alpha (mispricing relative to the CAPM) differs systematically by stock as also shown in Figure 4: since the market expected return also rises by the same percent, stocks with betas lower than the market beta of one appear to have positive excess returns relative to the CAPM (positive alphas) and stocks with betas higher than the market beta have negative alphas.

In Figure 4 we assume that $f(\pi) = \gamma(\pi - \bar{\pi})$. Then when inflation equals its long run average, $\pi = \bar{\pi}$, the regular CAPM holds. The slope of what is called the Securities Market Line (SML) is equal to the average market excess return. When inflation exceeds its long run average, $\pi > \bar{\pi}$, all mean excess returns exceed the required excess returns by the same percent. The new SML running from the origin through the now higher mean excess market return underpredicts the mean returns for assets with betas below one and overpredicts the mean returns for assets with betas above one. Cohen, Polk, and Vuolteenaho (2005) find some evidence of this, suggesting that the Modigliani-Cohn hypothesis explains some of the empirical shortcomings of the CAPM. Note however that the hypothesis does not explain why long-run average returns may differ from those predicted by the CAPM since $f(\pi) = \gamma(\pi - \bar{\pi}) = 0$ over the long run so that the CAPM should hold.

Application 2: *Intrinsic Valuation and the Required Market Return*

The question we ask here is how the mean return from investing in the S&P500 index is related to the required return for investment in the index based on its intrinsic valuation according to the Gordon Growth Model, as discussed in section 5(a). We use data available from Robert Shiller's website which includes monthly information from January 1871 to now on S&P500 index prices, dividends, and earnings.

To smooth temporary shocks that are better ignored in intrinsic valuation we consider a 10-year (backward) moving average of returns, dividends, and earnings. The important assumption is that the preceding 10 year history of earnings or dividends is a good estimate of future earnings or dividends. As earnings and dividends grow over time we adjust for the relevant trend over the previous 10 years by multiplying the moving average by $(1+g_{t,10})^5$. Figure 5 shows the moving average of the nominal index returns over time calculated from the dividend yield and capital gains of the S&P500 index, in relation to the moving average of inflation calculated from the Consumer Price Index. The average market return over the period starting in 1871 is around 10.0% and inflation is around 2.6% over the same period. Notice that the 10-year average of the market return is at no time far below zero, either in nominal or in real terms.

The historical values for the price-dividend ratio and the price-earnings ratio are shown next

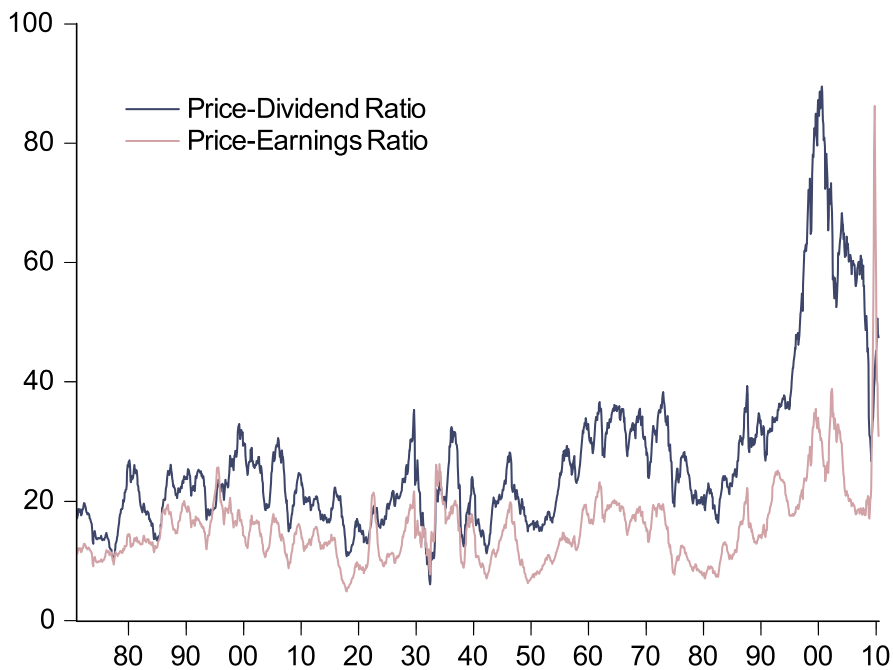

Figure 6

Intrinsic Valuation Ratios

in Figure 6 with the S&P500 index providing the price and the dividends and earnings provided by their respectice10-year trailing moving averages adjusted for growth. The average price-dividend ratio is 27.0 and the average price-earnings ratio is 15.5. Both ratios track eachother quite well (the correlation is 0.69) and the price-dividend ratio almost always exceeds the price-earnings ratio. For both series also the value of the ratios clearly outside of their regular range in the early 21st century could have been interpreted as a warning for the financial downturn in 2007.

The Gordon Growth Model implies that the required returns on the stock market index can be inferred from earnings and dividend yields as $k_{EY}^{E} = E_t(NI_{t+1})/P_t^{E}$, $k_{DY}^{E} = (E_t D_{t+1}/P_t^{E}) + g$ as discussed in section 5(a). To see how well the required market return inferred from the S&P500 earnings yields and dividend yield track the observed average S&P500 index return, we take the 10-year centered moving average of the index returns and compare it each point in time to the earnings (dividend) yield calculated by dividing the 10-year centered moving average of earnings (dividends) by the price index for the relevant month. Since real returns are more fundamental we subtract the appropriate inflation rates from the index returns and from the earnings yields, and add real growth

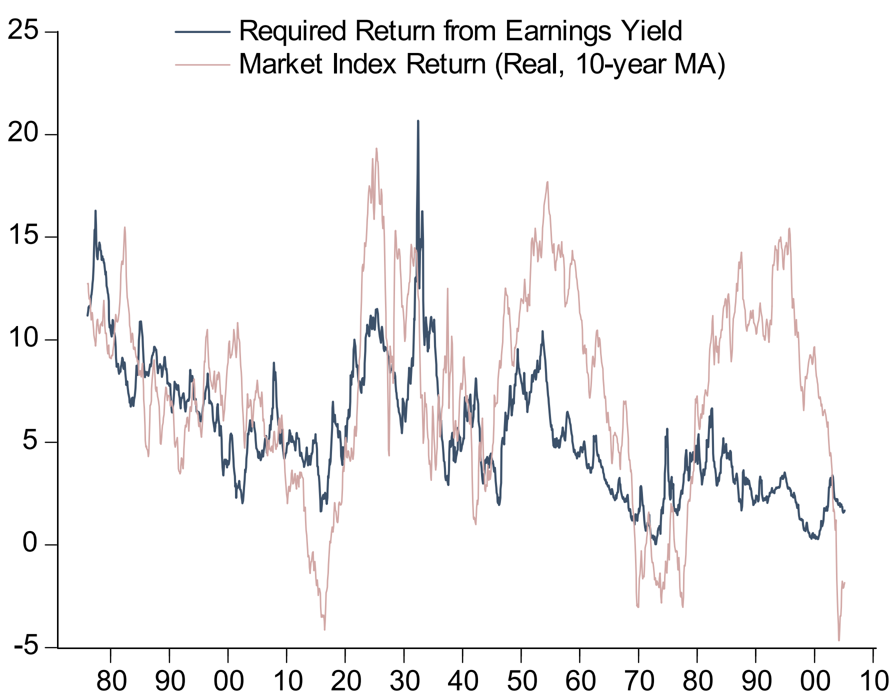

Figure 7

The Real Required Market Return Implied by the Earnings Yield

rates (calculated as the 10-year centered moving average of realized real dividend growth) to the dividend yields.

Figure 7 illustrates the match between the average return and the required return derived from the Gordon Growth Model based on the earnings yield. While average real S&P500 Index returns are 7.5% over the period from 1871 to now the average earnings yield net of inflation is only 5.5%. As the figure suggests, the two series are positively correlated with a correlation coefficient of 0.38. Figure 8 shows the required return based on the Gordon Growth Model using the dividend yield plus the real dividend growth rate, compared to the average market return. The average over the full period from 1871 for the dividend yield is 4.6% and for the real dividend growth rate is 1.2% so that the inferred required return is 5.8%. The dividend-yield-implied required return tracks the realized average returns a little better than those implied by the earnings yield, having a correlation coefficient of 0.58 with the moving average of the market returns.

Both intrinsic value measures, the earnings yield and the dividend yield, suggest that the historical average of S&P index returns has been 1.5-2.0 percent higher than implied by fundamentals. The numbers and comparisons provided here, are merely an illustration. The

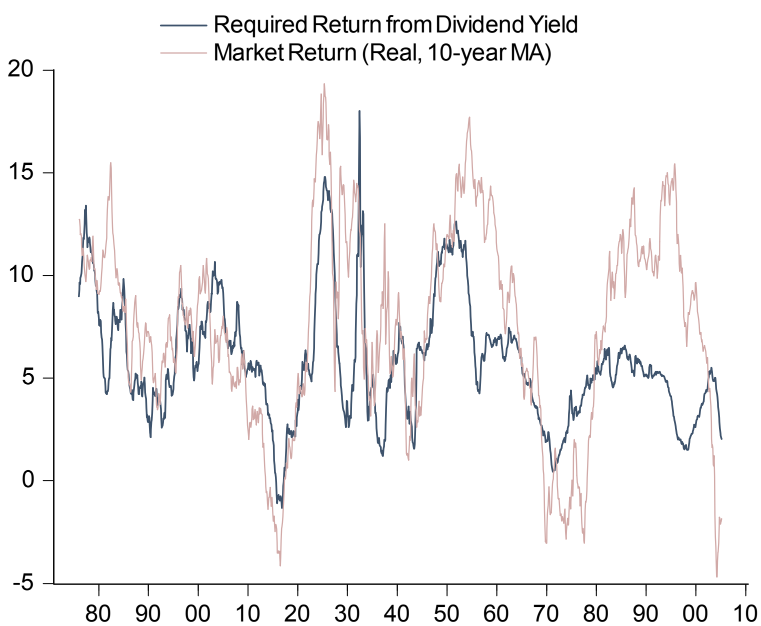

Figure 8

The Real Required Market Return Implied by the Dividend Yield

Dependent Variable: DJIA RETURN (5-year future average)
Method: Least Squares
Sample: 1920 2005
Included observations: 81 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	16.35956	1.599294	10.22924	0.0000
MTB	-2.061479	0.691650	-2.980524	0.0038

R-squared	0.101083	Mean dependent var	12.34940
Adjusted R-squared	0.089704	S.D. dependent var	8.155542
S.E. of regression	7.781155	Akaike info criterion	6.965668

Table 2. Forecasting Returns with the Market-to-Book Ratio

applicability of the intrinsic value ratios to the forecasting of return levels is not straightforward. For instance, Lamont (1998) shows that high dividend yields predict high market returns, but high earnings yields predict low market returns. (Possible explanations are that high dividends signal management's positive inside information, whereas high earnings yields simply reflect low stock price levels). The Gordon Growth Model, and other intrinsic valuation models, may also be used to forecast future equity returns using only information available in real time.

Application 3: *Book Value and Future Earnings*

The Dow Jones Industrial Average (DJIA) is an index of the 30 largest companies traded on U.S. stock exchanges. As it includes fewer companies than the S&P500 index it is sometimes easier to collect data for. In particular, we have book values of the index going back to 1920. Valueline provides data for the DJIA that include book values as well as earnings, dividends, and prices. See http://www.valueline.com/pdf/valueline_2006.pdf

The residual income valuation in section 4 relates market value to book value plus the present value of future income net of the opportunity cost of capital. From equation (4.20) dividing by book value gives

$$\text{(A6)} \qquad V_t^E/B_t^E = 1 + \frac{E_t[(NI_{t+1}/B_t^E) - \kappa_t^E]}{(1+\kappa_t^E)} + \frac{E_t[(NI_{t+2}/B_t^E) - \kappa_{t+1}^E](1+g_{t+1}^B)}{(1+\kappa_t^E)(1+\kappa_{t+1}^E)} + \ldots ,$$

in which g_{t+1}^B is the growth rate of book value. If we assume that this growth rate as well as the required return on equity and the expected return on equity $E_t(NI_{t+1}/B_t^E)$ follow independent random walks we can simplify equation (A6) to

$$\text{(A7)} \qquad V_t^E/B_t^E = 1 + \frac{E_t[(NI_{t+1}/B_t^E) - \kappa_t^E]}{\kappa_t^E - g_t^B} .$$

There are two easily testable implications from equation (A7) or equation (A6). First, higher market-to-book ratios forecast higher future returns on equity. Second, higher market-to-book ratios forecast lower future market returns on equity given that $\kappa_t^E = E_t(D_{t+1} + P_{t+1}^E - P_t^E)/P_t^E$ when markets are efficient.

It is straightforward to test these implications with forecasting regression of the form:

$$\text{(A8)} \qquad \mathit{future}\,(NI/B^E) = a_1 + b_1(P^E/B^E), \ \ \mathit{future}\,(\mathit{Returns}) = a_2 + b_2(P^E/B^E) ,$$

with $b_1 > 0$ and $b_2 < 0$. We take as the future values for the returns on equity and the market returns the average of their realizations over the following five years:

$$\text{(A9)} \qquad \sum_{i=1}^{5}(NI/B^E)_{t+i}/5 = a_1 + b_1(P^E/B^E)_t , \ \ \sum_{i=1}^{5} r_{t+i}^E/5 = a_2 + b_2(P^E/B^E)_t .$$

Using annual data for the DJIA and the associated annual earnings and book values for the period from 1920 through 2005 to test the implications we find significant positive forecast power as predicted and shown in Table 3.

So clearly the implications are confirmed statistically. Numerically, if the market-to-book ratio were to go from 1.0 to 2.0, future nominal expected returns would fall from around 14.3% to

Dependent Variable: EARNINGS-TO-BOOKVALUE
Method: Least Squares
Sample: 1920 2005
Included observations: 81 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.084931	0.008214	10.34013	0.0000
MTB	0.024058	0.003552	6.772623	0.0000

R-squared	0.367334	Mean dependent var	0.131730
Adjusted R-squared	0.359326	S.D. dependent var	0.049927
S.E. of regression	0.039963	Akaike info criterion	-3.577351

Table 3. Forecasting Earnings with the Book-to-Market Ratio

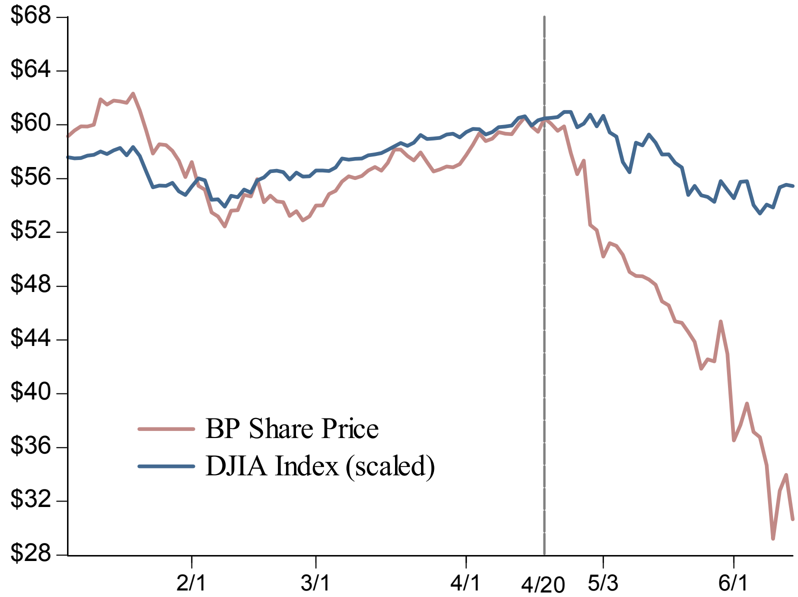

Figure 9
Market Estimate of Oil Spill Cost

12.2%. The same change in the ratio would raise the return on equity, which on average is 13.2% by around 2.4%. You can also interpret the relationship as required return expectations and the expectations of higher operating earnings being proxied by their future realizations determining current market prices as normalized by book values. In this case the two future average variables are on the right side and explain the market-to-book ratio. The result here is again as expected and is shown in Table 4 .

Dependent Variable: MARKET-TO-BOOK RATIO
Method: Least Squares
Sample: 1920 2005
Included observations: 81 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.481925	0.269880	1.785703	0.0780
DJIA RET (5-YR AVG)	-0.075160	0.011480	-6.547072	0.0000
E/B (5-YR AVG)	18.15486	1.875239	9.681354	0.0000

R-squared	0.591707	Mean dependent var	1.945286
Adjusted R-squared	0.581238	S.D. dependent var	1.257803
S.E. of regression	0.813947	Akaike info criterion	2.462491

Table 4: Explaining Market-to-Book Values

Application 4: *Learning from Financial Market Prices***.**

Financial markets often provide information about the costs and benefits of critical events How does the market evaluate the cost of the BP oil spill in the Gulf of Mexico? If we assume market efficiency and assume that BP (together with its venture partners Haliburton, Transocean, and Cameron International) will be helt responsible for all damages, we can infer the market's prediction of the present value of all costs by examining changes in BP's market value. To do so, one typically uses *event study* methodology.

The predicted present value of the cost (clean-up, litigation, lost opportunities) can be inferred by considering the change in market capitalization since the accident on April 20, 2010. Just examining the decline in share prices in not sufficient for two reasons. First, following the event study methodology, the decline must be considered relative to an appropriate benchmark. Second, given the Miller-Modigliani theorems, the change in the value of debt must be considered as well.

Figure 9 provides the price history of a share of BP stock from April 20 to June 14, 2010. The closing price on April 20 of one BP American Depository Receipt (ADR) traded on the NYSE at the close of June 14, 2010 was \$30.67, down from \$60.48 on April 20, 2010. As each ADR represents six shares of BP common stock and since 18.7 billion shares were outstanding at both dates, the market capitalization decreased from \$188 billion to \$96 billion. However, we should adjust for the change in price of an appropriate benchmark. Theoretically, the best benchmark would be a diversified portfolio with similar risk and no direct exposure to the event under consideration. For convenience here we take the DJIA as it provides a diversified market average of similarly-sized firms. We normalize the DJIA index by multiplying each of its values by the ratio of the BP price to the DJIA index at April 20. As Figure 9 shows, the index fall also over the same period (supposedly for reasons unrelated to the oil spill), from \$60.48 to \$55.44, a drop of 8.3% that in all expectation would have been similar for BP had the oil spill not happened. So the proper benchmark price for BP is \$55.44, and the loss in share holder value is (18.7/6) billion times (\$55.44 - \$30.67) is \$77.2 billion.

In addition, the market value of BP's (long-term) debt fell over the same period. The decrease in the value of debt is as a result of the limited liability of the BP shareholders: given a range of possible realizations for the costs to BP, the shareholders end up not paying for all of it in states in which it goes bankrupt. The remaining costs in these bankruptcy states will be (mostly) absorbed by the debtholders, and can be measured by the decrease in prices of BP's corporate bonds. Rather than considering the prices and numbers of BP bonds outstanding we'll take a shortcut to obtain a rough estimate. Based on first-quarter 2010 results (from Value Line), the book value of BP's long-term debt was $25.5 billion. Given an approximate duration (average term to maturity) of these bonds of a little above 3 years we examine the yield to maturity of the BP bond maturing November 2013, which is 8.35% on June 15. Based on equation (3.3) we can infer the market assessment of the probability of BP bankruptcy by subtracting the yield to maturity of a (default risk free) Treasury Note with 3 years left to maturity, which is 1.30% on June 15. Hence, the implied probability of bankruptcy is 7.05% *per year* for the three years until 2013. In the period before April 20 the implied probability of bankruptcy was a stable 0.48% per year. Roughly, the decrease in the value of debt then is around 3 x 6.5 = 19.5% which amounts to an amount of $5.0 billion.

The total market-implied cost estimate becomes $82.2 billion. On June 14, U.S. senators demanded that BP put $20 billion in escrow to meet compensation costs, which is in addition to $14 billion in civil penalties payable under U.S. environmental law. A few weeks earlier Credit Suisse had estimated a maximum total bill of $37 billion – costs of cleanup plus liability for economic damages. As the recent news about the extent of the spill has been discouraging, the market estimate of the cost does not appear to be out of line. Recent implied probabilities of bond default have been increasing accordingly. BP bonds imply probability of default of around 8.5% for the coming year, and slowly decreasing to below 4.0% a year after five years. Based on the bond maturing March 2019, the cumulative probability of bankruptcy by the time of maturity is in excess of 35% on June 14. Credit Default Swaps tell a similar story, with the cost on June 15 of insuring against BP default at 506 basis points a year for BP's five-year bonds.

Exercises*: Asset Valuation*

1. Consider the Gordon Growth Model with a finite stream of cash flows. Specifically, assume that cash flows grow at a constant rate until time T and are zero after time T.

   (a) Derive the Present Value of the stream of payments under discrete compounding.

   (b) Explain why the Present Value of the stream of payments under continuous compounding, and if a fixed payment X is paid continuously through time, is given as:
$$\int_0^\infty e^{-rn}(e^{gn}X)\,dn = X\int_0^\infty e^{(g-r)n}\,dn = \frac{X}{g-r}[e^{(g-r)n}]|_0^\infty = \frac{X}{r-g}\,.$$
   Here we obtain essentially the same result as in the discrete-compounding case except that all constants need to be interpreted as continuous-compounding-based.

2. Derive the Gordon Growth Model of equation (2.14). Explain how inflation affects the present value in this model. Assume that projected inflation is 3%, projected stock returns are 10% and the growth rate of dividends is projected to be 5%. Calculate what the price/dividend ratio (based on current price and current dividends) should be according to the Gordon Growth Model.

3. Prove that the n-period discount factor at time t, β_t^n [defined as in equation (1.1) but with $c_t = c_{t+i}$ for all $0 \le i \le n-1$], is equal to the product of one-period discount factors: $\prod_{i=0}^{n-1} \beta_{t+i}$.

4. Consider the stock with value V of a company that each year faces the same earnings distribution X. The earnings are all paid to the stock holders. There is a probability F in each year that the company fails, in which case of course its stock becomes valueless.

   (a) Explain intuitively that the value of the stock can be written as: $V = \dfrac{E(X) + (1-F)V}{1+\mu_X}$.

   (b) Contrast the expression of V here with equation (4.3).

   * (c) Assuming that the CAPM applies, solve for V and obtain the Certainty Equivalence form of adjusting for risk. [Hint: define $z = 0, 1$ to represent realized earnings for the period, with $z = 0, 1$ depending on whether or not the company fails].

5. For the standard Gordon Growth Model, with a given constant required return and constant growth rate of dividends, given in equation (4.3):

   (a) Derive the expected return.
   (b) Discuss how the Gordon Growth "asset pricing" Model compares to the CAPM asset pricing model.
   (c) Assume that the CAPM holds for two firms with identical betas but different growth rates. For the two firms compare the expected returns and their capital gains rates.

6. To relate the Gordon Growth Model to earnings, explain what assumptions on net income over time NI_t and investment over time I_t must be made to derive the Gordon Growth Model?

7. Given the Ohlson Model and the particular ARMA formulation for residual income in section 7:

   (a) Derive the value of equity stated in equation (7.1).
   (b) Based on the value of equity equation, derive that the expected return is equal to the required return when the book value of equity is equal to the market value of equity.

8. In their projections, Value Line (http://www.valueline.com/pdf/valueline_2006.pdf) assumes long-run values for price-dividend ratios of 30, price-earnings ratios of 15, and book-to-market ratios of 1/3 for the 30 companies in the DJIA.

   (a) Calculate the long-run average growth rate of dividends and the average required return on equity for the 30 Dow Jones companies implied by the Gordon Growth Model given the long-run values assumed by Value Line.
   (b) Use Equation (5.11) to infer the long-run growth rate of dividends implied by the long-run values assumed by Value Line.
   (c) Explain the contradiction between the numerical long-run growth rates inferred in items (a) and (b).